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ABSTRACT

The study was concerned with the formation of grouPs of students and specifically addressed the problem: Can a computerized Procedure be developed which assigns students to instructional groups, which maximizes the homogeneity of these groups when this homogeneity is based on relevent student learning characteristics, and which takes account of realistic administrative constraints? The mathematical procedure developed involved utilizing computer technology in its implementation. It aimed to facilitate, in part, the management of Individually Guided Education (IGE). Four algorithms were designed, each involving the fitting of a homogenizing procedure within the framework of the administrative constraints of the problem. The algorithm which proved to be most effective was the one which initially assigned students to groups, matched group sizes with skills, allocated eligible students to these groups to maximize their homogeneity and then applied other administrative constraints. The effectiveness of this algorithm was further assessed by comparing its recommended groupings with teacher generated groupings when both were subject to the same constraints. The computerized procedure produced more homogenous groups than did the teachers and an equivalent number of students were omitted. User perceptions of the efficiency and effectiveness of the procedure were also obtained. It is claimed that the procedure developed warrants further evaluation. (RC)

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TECHNICAL REPORT NO. 399
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numerical procedures in the optimal grouping of students for instructional purposes

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SEPTEMBER 1976

WISCONSIN RESEARCH
AND DEVELOPMENT
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Technical Report No. 399
Part 1 of 2 Parts

NUMERICAL PROCEDURES IN THE OPTIMAL GROUPING OF STUDENTS FOR INSTRUCTIONAL PURPOSES

by

Brian F. Lawrence

Report from the Project on
Computer Systems for IGE

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Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

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FUNDING

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This dissertation is dedicated to Seven Little Australians and their mother, the author's wife.

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ABSTRACT

This study was concerned with the formation of groups of students and specifically addressed the problem: Can a computerized procedure be developed which assigns students to instructional groups, which maximizes the homogeneity of these groups when this homogeneity is based on relevant student learning characteristics, and which takes account of realistic administrative constraints such as eligibility for group membership, sizes of groups, and numbers of groups?

The procedure developed to solve this problem was mathematical in nature and involved utilizing computer technology in its implementation. It aimed to facilitate, in part, the management of a particular individualized program of instruction, namely Individually Guided Education (IGE).

Based on an initial survey of clustering techniques including hierarchical techniques, optimization-partitioning techniques, density-seeking techniques and clumping techniques, a decision was made that the optimization-partitioning techniques applied most directly to the problem being studied. This set of techniques was further surveyed in terms of complete enumeration, implicit enumeration procedures and heuristic procedures which yield local optimal solutions. Despite their disadvantage of yielding sub-optimal solutions, the heuristic partitioning procedures were considered to most closely meet the requirements of the problem.

Four algorithms were designed, each one involving the fitting of a homogenizing procedure within the framework of the administrative constraints of the problem. The homogenizing procedure employed was the Forgy minimum variance partitioning procedure modified by using a proportional division method for selecting seed points and the weighted Euclidean metric as a measure of similarity. The four computer based procedures were evaluated on the basis of their performances on a set of tests which involved varying the parameters of the grouping situation, such as the data on learner characteristics, data on group eligibilities, the number of groups formed, the sizes of the groups, and the single or multiple assignment of instructional topics to groups.

Two equally important criteria were used in the choice of the most effective of the four algorithms--the homogeneity of groups measured on selected learner variables and the number of students omitted from the groups. The algorithm which proved to be most effective was the one which initially assigned instructional topics to groups, matched group sizes with skills, allocated eligible students to these groups to maximize their homogeneity and then applied other administrative constraints.

The effectiveness of this computer based grouping algorithm was further assessed by comparing its recommended groupings with teacher generated groupings when both groupings were subjected to the same constraints. In the comparison performed, the computerized procedure produced much more homogeneous groups than did the teachers and an equivalent number of students were omitted. The profiles of the groups formed by the two methods were noticeably different as

determined by the differences in the means of the learner characteristics for each group, a ratio of agreement and the phi coefficient of association.

User perceptions of the efficiency and effectiveness of the computerized grouping procedure were also obtained. The computerized grouping procedure was perceived to be much more efficient in terms of time spent by users in the grouping process than a manual procedure and more efficient than a semi-automated procedure used by the teachers. Respondents, however, mainly gave median ratings of the computerized procedure's success in maximizing the homogeneity of the groups and minimizing omissions from the groups.

The evaluation of the computerized grouping procedure performed as part of this study can only be considered as preparatory to a more comprehensive examination of the effectiveness and efficiency of the computerized grouping procedure. Despite this limitation, it is claimed that the procedure developed warrants this further evaluation.

APPROVED D W Spuck

DATE August 13, 1976

CHAPTER I

FOUNDATIONS OF THE PROBLEM

Introduction

Attempts at improving the instructional and learning processes have frequently emphasized individualized instruction which Suppes (1966(a), page 207) defined as "an adaptation of an educational curriculum in a unique fashion to individual learners each of whom has his own characteristic initial ability, rate and style, to provide him with a successful learning experience." Systems of individualized education are oriented towards individual abilities, interests and needs and take account of differences in learning styles, instructional levels, rates of progress as well as in instructional strategies. Wright (1972, page 77) identified similar defining characteristics of individualized instructional programs when, on the basis of a comprehensive review of the relevant literature, he recognized these programs as providing for differences in

(1) learning rates,
(2) learning styles,
(3) student participation in goal setting,
(4) student participation in determining learning sequences,
(5) student grouping based on student characteristics, desires, and needs.

Within some individualized instructional programs, student groups are established for specific purposes and then dissolved when these purposes are achieved (Martin and Pavan, 1976, page 311). A group comprises students who at a specific time have common concerns, needs, interests or plans and may be formed for students to share a common meaningful experience, to participate in specific activities, or to attain skills not available in another mode. Instructional resources such as team teaching, television, film, slides, language laboratories and self-education programs are readily adaptable to such groupings and were considered by Belt and Spuck (1975, page 7) to be realistic solutions to the problem of adjusting instruction to the individual differences in students.

In the past, groupings of students for instructional purposes were often based on a very small number of parameters (e.g., age and intelligence measures), considered permanent, and applied uniformly to a wide range of curricular subjects. Such groupings have been conclusively shown to have little effect on reducing the degree of heterogeneity of the group and also to have deleterious effects on the motivation, self-image and achievements of the students (Heathers, 1969, page 14 and Westby-Gibson, 1966, page 10). However, there can be little argument that groups formed for a specific purpose can reduce the differences among individuals when these differences are in the area identified by the dependent variable used to form the group. Equally obvious is the strong likelihood that variation in other student variables may be increased as a result of the grouping. This raises the question of whether or not educators can identify a specific set of dependent variables that pertain to a specific content area. If not, homogeneous grouping will continue to be suspect as an attempt to provide for individual differences in learning.

The general literature on individualized education has identified several learner characteristics which should be taken into account when forming groups for instructional purposes (McNamara and Spuck, 1975, page 6; DeVault and Kriewall, 1970, page 416; Heathers, 1969, page 21; and Suppes, 1966(a), page 207).

These characteristics typically included learning style, learning rate, interest level, and deficiencies in knowledge base. To be useful in numerical grouping procedures, these characteristics need to be measured. It is expected that where measurement of such learner characteristics is possible, it will be at the ordinal or interval levels. Because grouping is essentially based on degree of similarity, it can be further expected that these levels of measurement will result in adequate data in the measurement of interstudent similarity. Instruments useful in measuring learning styles and interest levels are often of the self report or observational types and measures of learning rate and deficiencies in knowledge base are available as a result of periodic testing done as part of the instructional program. What is not considered in the literature is the effect on the composition of the instructional groups and hence on individual performances of using different combinations of data on learner characteristics and learner past performance. Not only should grouping procedures permit the use of various relevant variables, but their selection should be based on evidence of their singular and collective effects on the achievement of the group members.

Some schemes of individualized instruction which attempt to meet the needs of individual students by bringing together learners having common attributes base these groupings upon teacher opinions, test scores or some subjective assessment involving both test scores and student characteristics. Because of the subjective nature of the procedures, the limited range of student characteristics considered when forming the groups and the inefficiency of the methods used, it cannot be expected that the degree of homogeneity of the groups will be optimal. Consequently, it is unlikely that the goals of such programs of individualized education will be met by so forming instructional groups.

The Individually Guided Education (IGE) scheme presently being utilized in over 2,000 American schools attempts to meet the needs of individual students by establishing appropriate instructional groups. Klausmeier, Quilling, Sorenson, Way, and Glasrud (1971, page 18) report that in IGE rather than having one teacher who is more or less responsible for 20 to 35 students, three to five teachers and other teaching aides work as a team to guide the education of 100-150 students; these teachers, aides, and students make up an instructional unit. Such factors as the nature of the instructional material, and student and teacher characteristics are involved in identifying instructional groups and establishing group sizes. Thus, while group teaching is characteristic of traditional classrooms and students working by themselves is a characteristic of "file-folder" approaches to individualization, instruction in IGE takes place in various size groups, large group instruction to individual work, with the small to medium sized group being the most common. Proponents of IGE strongly believe that such group interaction is the most effective use of learning certain concepts (Belt, 1975, page 15).

Essential to the functioning of such programs of individualized education is the teacher's ability to cope effectively with the large volume of information required in the management of these programs. Monitoring the progress of students and deciding upon optimal instructional objectives, tasks, and organization becomes an extremely complex and difficult endeavor. Experience in working with these complex programs has led to an increased awareness that computer-based management information systems are essential to their effective implementation and operation (Baker, 1971, page 51 and Spuck and Owen, 1975, page 2). Accordingly, the Wisconsin System for Instructional Management (WIS-SIM) is being developed as a generalized scheme of computer support for the instructional management needs of IGE schools (Spuck, Hunter, Owen and Belt, 1975, page 21). It is within the framework of WIS-SIM that a computerized grouping procedure fits.

Based on considerations of schemes of individualized instruction such as IGE, it seems that the grouping procedures involved should be economical of teachers' time and should have the capability of forming mutually exclusive groups,each of which has members who are maximally similar with respect to specified characteristics related to instructional needs.

This study was concerned with the formation of groups of students and specifically addressed the problem: Can a numerical procedure be developed which assigns students to instructional groups, which maximizes the homogeneity of these groups when this is based on relevant student learning characteristics and which takes account of realistic constraints on resources such as numbers of groups and sizes of groups?

The procedure developed to solve this problem was mathematical in nature and involved utilizing computer technology in its implementation. The study itself comprised the development, application and evaluation of the procedure.

Grouping students for instruction is but one aspect of the instructional program about which decisions are made by school personnel. These decisions can be expected to reflect the educational philosophies of those involved in their making and are specifically influenced by earlier decisions made in the areas of diagnosis of student needs, formulation of objectives and selection and organization of content and learning experiences. Therefore, the further expectation was that the solution to the above problem would be based on an analysis of the specific educational environment within which the solution was to be applied. The procedure developed in this study aimed to facilitate, in part, the management of individualized programs of instruction, the features of which are now described to support the significance of the problem as well as its further clarification.

Significance of the Problem

The significance of the problem derived out of a further consideration of some features of individualized instructional programs alluded to in the introduction. In particular, the problem's significance was supported by (1) the central role of grouping practices in individualized instructional programs, (2) the need for providing instructional decision-makers with more relevant information on which to base groupings, and (3) the need to provide more efficient and effective procedures in the formation of the groups.

Brueckner and Grossnickle (1968, page 89) pointed out that individualization of instruction does not imply that the instruction must be so organized that each individual works by himself on a specific task, but that actually certain capacities of the individual are stimulated by association with others. If this point of view is accepted, one does not reach the conclusion that the wide range of differences found in typical classrooms makes grouping impractical.

Grouping and regrouping within a classroom for instruction in particular subjects is an accepted and recommended practice (Martin and Pavan, 1975, page 311). Wright (1972, page 76) suggested that student grouping will continue to be an acceptable practice in educational institutions, but like Martin and Pavan recommended that groups be formed for specific purposes and maintained only so long as these purposes remain viable. Given the validity of these recommendations and the diverse and comprehensive curricula of modern elementary schools, one can expect that individual students will belong to a variety of groups even over a relatively short period of time. Keeping track of even a single student becomes a considerable clerical task, and one that can prevent the classroom teacher from spending more time on the important matters of instructional programming and teaching. Consideration for this feature of individualized programs draws attention to the logistical problems involved in manually forming and reforming groups for different instructional purposes. Most of these problems relate to routine matters of record keeping and information retrieval, difficulties which are accentuated by both the scope and amount of student related information considered to be essential for the formation of instructional groups.

In the past, most instructional models for optimizing instruction have utilized limited performance data for adapting or individualizing the instructional process. However, there is good reason to believe that a truly adaptive instructional decision model should incorporate affective as well as cognitive response data in order to fully optimize the instructional process for the individual learner (McCombs, Eschenbrenner, O'Neil, 1975, page 47). The affective domain typically deals with attitudes, values, interests, and personality traits. In addition, it includes motivational traits (anxiety and curiosity). Considered important also are the students' current reactions to instructional variables such as content, presentation style and difficulty level. The use of all or even some of this information in the grouping process requires both summarization and computerization. A system capable of synthesizing large amounts of relevant information while being efficient of teachers' time is a requirement of individualized instructional programs. An automated grouping procedure which is part of a generalized automated instructional management system is likely to prove more useful than manual systems currently employed. Generalized automated instructional management systems such as WIS-SIM possess the capability of inclusion of a grouping procedure which takes account of learner characteristics such as those referred to above. The WIS-SIM model in particular has been conceptualized so as to take into account for instructional purposes a wide range of both subjective and objective information such as aptitudes, learning style, and learning handicaps.

Traditionally, grouping procedures have been a subjective result of some objective measurement process. Student records in various subject areas are obtained from a variety of sources; for instance, previous grades, teacher evaluations and standardized tests. The administrator then sets a few basic decision rules, and groups or clusters students on this basis. The effectiveness of this procedure is open to question: Are the resultant groups in any sense maximally homogeneous? Given the desirability of using a greater volume of more relevant data in the grouping process, both the impracticality and the subjective nature of a manual grouping process are likely to prevent the formation of groups which are sufficiently homogeneous to attain the purpose of the grouping. A more objective grouping procedure may result in a greater degree of homogeneity with the use of statistical and computer technology. These technologies are presently available and await their adaptation in the implementation of individualized programs of instruction.

Not only should systems for the grouping and regrouping of students take account of various relevant learner characteristics but they must operate within realistic limits on resources. Consideration of constraints such as the number of groups, the sizes of these groups and the prior performances of individual students in particular instructional programs not only makes the problem more relevant for schools implementing programs of individualized instruction, but also makes the problem's solution more complex.

The solution to the problem of efficient and effective formation of groups for instructional purposes appeared to warrant the use of statistical and computerized procedures. Such procedures have the capability of providing for (1) the efficient storage and processing of student related data, (2) the availiability of grouping recommendations upon request and (3) a high degree of homogeneity in group membership.

Because computerized management procedures had found little application in the administration of programs of individualized instruction and the use of numerical grouping procedures even less so, it was considered that a successful joint implementation offered the possibility of their more extensive use both in the grouping of students for instructional purposes and also in other educational problems where classification is necessary.

The precise formulation of the problem was dependent upon an analysis of the educational environment within which it was set. This analysis is presented in the next three sections, the first of which is concerned with the role of grouping in individualized instructional programs.

Grouping Within Individualized Programs of Instruction

This section seeks to clarify the purposes of grouping students, to identify the components of acceptable grouping practices and on the basis of this analysis to derive an initial set of criteria to be met by an automated grouping procedure.

A philosophy of grouping is closely related to attitudes towards education and individual differences; thus, the position one takes on grouping is primarily dependent upon one's basic conceptions of the nature of the individual and of the purposes of education. Given that this is so, grouping procedures should be brought into harmony with all the major objectives of education. So conceived, the problem of grouping students for instructional purposes is basically as broad as the accepted objectives of education.

Yates (1966, page 97) referred to grouping as a device for achieving a better fit, congruence or relationship between students and something else. This something else could be the teacher, the task or activity, some set of common purposes, or a generalized social role. All schools group students for instructional purposes. It is not only necessary for practical purposes, as in the sharing of scarce learning resources, but desirable for educational reasons that students be grouped for instruction for a significant portion of their time at school.

The question of grouping is not one of desirability but one of determining what system of assigning pupils and teachers to instructional spaces will best facilitate the accomplishment of school-wide goals and instructional objectives for learners.

Grouping procedures can themselves be usefully subdivided into two types, each derived from the purposes for which the groups were initially formed. One type has as its purpose the teaching of subject skills on the level of the child's needs and the other grows out of an awareness of the need of the students in a democracy for actual practice in living and working together efficiently and happily. The former groups may be referred to as being homogeneously formed with respect to some goal, the latter as being heterogeneously formed. To expect perfect homogeneity of individuals within a group is unrealistic and can only be attained when group members are identical in terms of a continuous scale with maximum homogeneity of group membership as one end point and minimum homogeneity or maximum heterogeneity as the other end point.

The essential difference between the two basic types of groupings is therefore the degree to which the similarity of group members is emphasized when this similarity is measured in terms of chosen learner characteristics or needs. Homogeneous grouping refers to the organization of students on the basis of student similarity on one or more specific characteristics. The criterion for this classification may be, for example, age, sex, IQ, achievement or a combination of these or other variables. Alternatively, heterogeneous groupings include a diverse mixture of children who differ on one or more variables. This practice results from the hypothesis that in many subjects, varied levels of maturity and experience may contribute more to the learning process and that interaction of varied age groups may contribute to social growth and understanding as well as to academic growth. Consequently, pupils are sometimes grouped heterogeneously as a matter of deliberate policy. In the United States, heterogeneous grouping owes much to John Dewey's influence. He maintained that a school class should ideally be a society miniature, containing the same merging of social classes and levels of ability as one found in the adult community. This special significance of grouping in a democracy was recognized by Petty (1953, page 17) and Hildreth (1962, page 286) who stated that skill in group living is not learned by chance but is definitely planned for in the elementary school and that group activities help children learn the value of orderly procedure, taking turns, working with a leader, contributing to and sharing in a common cause. Given the validity of these arguments, a complete grouping procedure should facilitate the creation of groups sufficiently heterogeneous to meet the purposes of the grouping as specified by those responsible for the formation of the groups. Such purposes may be best attained by the random selection of group members. The random assignment of students to groups is of course the antithesis of the creation of homogeneous groups of students, which topic is next discussed.

A common assumption in the formation of homogeneous groups is that a teacher can more readily adapt instruction to differences among students when the range of differences within a class is reduced. This is so because group teaching becomes more manageable when the members of a group have more characteristics in common. Such grouping makes it possible for the teacher to adapt methods and materials more closely to the level most appropriate for the students.

Teachers often subdivide their classes to facilitate instruction. Subgrouping has been more apt to occur in heterogeneous classes than in ability grouped classes since teachers have employed it to accomplish within group homogeneity when this is based on ability or achievement data.

Such subgrouping is most common in elementary schools and is used most frequently with instruction in the skill areas of reading, spelling, and arithmetic. It is also used when conducting project activities in science and social studies.

DeVault and Kriewall (1970, page 418) noted that mathematics seems to be one area of study which can be undertaken without major concern for the role of one's peer group. They claimed that the degree of participation in this kind of group structuring activity would seem to be an important adjustable parameter of the fully individualized learning environment. Forming homogeneous groups of students for instruction in reading is a common practice in elementary schools and the subject has received considerable attention in the professional literature. Petty (1953, page 39) noted the primary consideration in homogeneously grouping students for instruction in reading as being: (1) the instructional reading level of the students, (2) the general interest level of the students, (3) the specific needs of students embracing language skills, concepts, critical reading, social adjustment or other learnings varying all the way from word recognition to reorganization of information.

Acceptance of the rationale for forming maximally homogeneous groups of students for specific instructional purposes especially as these occur in the skill subjects in the elementary schools, required its inclusion in a set of criteria to be met by an acceptable grouping procedure. The first such criterion recommended the formation of homogeneous groups.

Criterion I. A numerical grouping procedure should provide for the creation of maximally homogeneous groups.

In the 1920's and 1930's many studies attempted to compare heterogeneous with homogeneous groupings. Heterogeneous grouping was commonly based on a division by chronological age; homogeneous grouping, on standardized tests of mental ability, often combined with measures of achievement. The outcomes of the related research are inconclusive and indefinite (Heathers, 1969, page 2). What now seems to be clear is that many factors other than the heterogeneity or homogeneity of the groupings were involved. What the studies seemed to show us was that teaching approximately the same subject matter in approximately the same way produced approximately the same results whether ability grouping was used or not. These grouping practices were often based on the assumption that by choosing the proper grouping criteria, the variation of individual differences within the classroom group could be reduced significantly, and by so doing, enable teachers to teach more effectively. The facts are that whatever the criteria, a reduction in variability of a group by more than twenty percent is unlikely (Heathers, 1969, page 10). Neither does ability grouping alone enable teachers to create optimum learning situations. So called ability grouping tends to set arbitrary patterns which restrict rather than encourage pupils to make full use of their individual potential. Grouping on the basis of ability measures to create fixed groups over a range of subject areas has generally been superseded by individualized instructional programs in which flexibility of the grouping arrangements reflect the more individualized goals of the programs.

Consideration of the above argument suggested a further criterion of acceptability for a grouping procedure:

Criterion II. A numerical grouping procedure should permit the storage of diverse data from which selections can be made to meet different instructional purposes.

In the sixties and early seventies a number of educational innovations were tried in classrooms across the United States. Among the best known are nongradedness, team teaching, vertical or heterogeneous grouping of students, and the use of open space school building design.

The most widespread plan for continuous individualized progress in school is the nongraded school. The staff utilization plan that has done most to implement the nongraded school is "cooperative teaching." This includes any plan whereby two or more teachers work with the same group of students. In this form of associative teaching, several teachers have joint responsibility for a common group of students. The various patterns of cooperative teaching all make possible flexibility in grouping practices. They assume a way of working that permits the team to make decisions about group size and composition as well as teacher roles that contribute to the most effective learning situation. They permit teachers to utilize whatever size and type of group which seem warranted for given kinds of instruction.

Trends in school building design also reflect changes in organization towards nongraded schemes of individualized instruction. There appear to be two main trends away from the conventional type of school planning: (1) a building which deliberately caters to a very wide range of group sizes and group activities, (2) a building which is not based on any particular teaching method but is adaptable during its use to a wide range of learning activities. The potential range of spaces required for learning was broken down into five categories by Yates (1966, page 113): those required by

(1) an individual child;

(2) small groups of two or more students, but clearly below the conventional class size;

(3) a group which is at or about the conventional class size;

(4) groups which are noticably larger than the conventional class size, of say fifty or more;

(5) a large assembly which may range up to the whole enrollment of the school.

With specific regard to instructional grouping, the literature identified four basic grouping patterns--independent study, one to one, small group, and large group. All these need to be considered by staff teachers as they plan instructional activities for students. Consideration of student learning styles, needs and interests demands that each of these groups be available at some time for all students.

Consideration of the above factors led to the statement of a third criterion of acceptability of a numerical grouping procedure:

Criterion III: The grouping procedure should permit the formation of groups, the sizes and numbers of which can be specified by those responsible for the formation of the groups.

Grouping procedures recommended for use with one prominent nongraded individualized instructional program--Individually Guided Education (IGE) are now considered.

Manual Grouping Practices in IGE

Individually Guided Education (IGE) is a comprehensive nongraded system of instructional programming, team teaching, differentiated staffing and decision making. IGE, in part, attempts to meet the needs of individual students by establishing appropriately sized instructional groups. The grouping procedures typically recommended in IGE schools include the utilization of data from three basic sources:

(1) cognitive data from tests, observations, performances, and work samples,

(2) personal data on learning styles,

(3) affective data such as interests, attitudes, and motivational levels (Sorenson, Poole and Joyal, 1976, page 37).

The recommended IGE process of grouping students for instructional purposes is complex but generally proceeds as follows:

(1) identify the objectives;

(2) analyse the cognitive, personal, and affective data obtained on the students;

(3) assign students to groups, and groups to physical space;

(4) assign unit staff to groups;

(5) monitor and evaluate student progress.

Typically, teachers assigned to the unit of between 100-150 students specialize in handling the various data (e.g., one teacher may be responsible for assembling data on pre-tests, another on learning styles, another on interests, and so on. With the responsibility for preparation of summary data delegated among several teachers, Sorenson, Poole and Joyal (1976, page 38) divided the grouping problem into three successive stages:

(1) Form groups according to assessment data ;

(2) Reconsider groups formed in (1) and/or form subgroups from groups formed in (1) on the basis of learning style;

(3) Reconsider groups and/or subgroups from (2) above on the basis of interests, attitudes, and motivational level.

Sorenson, Poole and Joyal (1976, page 38) also provided the following illustrative example of the grouping process. The illustration involves grouping one hundred and twenty 9-12 year old students for instruction in a science topic (vertebrates).

The teacher who had assumed the responsibility for summarizing the assessment data (pre-test results) has already placed the 120 children in the unit into four groups and has duplicated the names of the students. These appeared as follows:

(40 students) 1. Students who do not meet the mastery level (90 percent) for any of the five classes of vertebrates and need work on all of the material.

(30 students) 2. Students who need work on three of four classes: mammals, fish, amphibians, and reptiles.

(40 students) 3. Students who need work on two of three classes: mammals, amphibians, and reptiles.

(10 students) 4. Students who meet the mastery level on all five classes and do not need basic work on vertebrates.

The initial grouping step can be expected to take approximately five minutes of meeting time.

The second step in grouping is to reconsider the initial groups formed on the basis of assessment scores. In the illustration this reconsideration is based upon learning style, but need not be. The teacher "specialist" on learning style and media preference adds the data on learning style which includes (1) attention span—continuous, irregular, short bursts; (2) sound tolerance—low, medium, high; (3) group size—alone, one-to-one, small, other; (4) assignment type—teacher or student-selected; (5) perceptual strengths and styles (preferred media). These 40 students fall into three subgroups as indicated below. The teacher specialist on affective data looks at the needs of these 40 students and compiles the entire list at once. Doing this, the students who do not meet the mastery level for any of the five classes of vertebrates are subgrouped as follows:

(10 students) Group A. (continuous attention span)
(low tolerance for noise)
Learning Style (small group (3-13 students))
(self-selected assignments)
(a combination of printed and audiovisual materials)

Affective (medium interest)
(medium motivation)
(low to medium attitude toward science)

(17 students) Group B. (short bursts of concentrated effort)
(tolerated distant noise well)
Learning Style (small group)
(teacher-selected tasks)
(mainly audiovisual and activity-oriented materials)

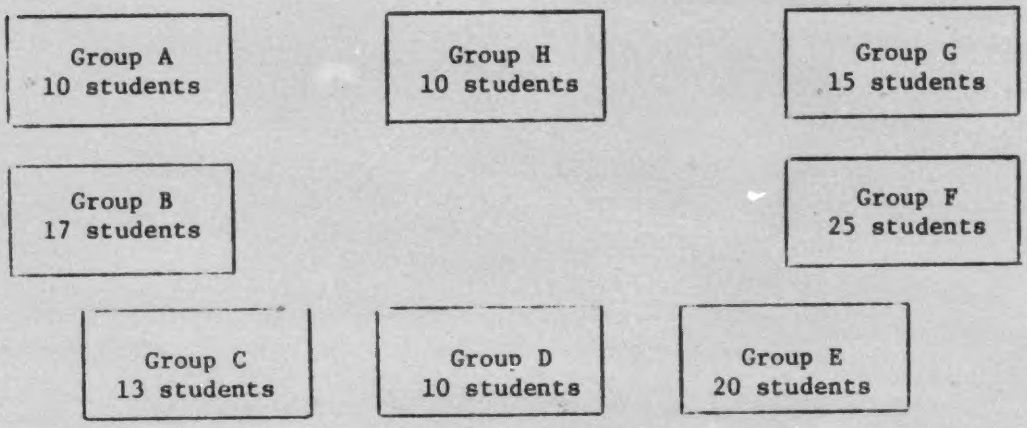
Affective
(low interest)
(low motivation)
(medium to low attitude toward science)

(13 students) Group C. (irregular attention span)
(high level of activity/noise)

Learning Style
(one-to-one and small group)
(teacher-selected tasks)
(high interest, audiovisual materia and activities)

Affective
(low interest)
(low motivation)
(low attitude toward science)

The three remaining initial groups formed on the basis of assessment are reconsidered in the same manner as above. Reconsidered groupings are indicated in the diagram below.


Group A
10 students
Group H
10 students
Group G
15 students
Group B
17 students
Group F
25 students
Group C
13 students
Group D
10 students
Group E
20 students

School areas are then assigned primarily on the basis of groups' needs relative to: (1) noise level; (2) heavy use of audiovisual equipment; (3) facilities for one-to-one and independent study work. Sorenson, Poole and Joyal (1976, page 41) reported that the assignment of students to groups and groups to spaces can be expected to take approximately 35 minutes.

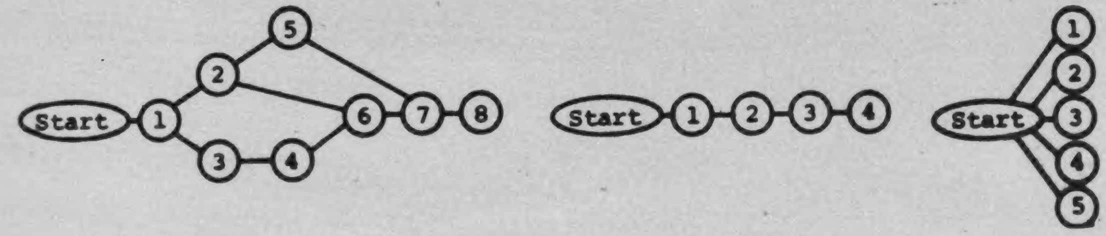
The purpose of the above grouping scheme seems clear as do its procedures. However, its efficiency and effectiveness remain dubious. Its objective is to form homogeneous groups when similarity is measured on such factors as pre-test results, learning styles and interest levels. The procedure for forming groups is characteristically sequential with subgroups being formed from within previously determined groups. It is a decomposition method, the composition of the final groups being independent of the order in which the factors are considered. The number of groups to be formed is not a prior constraint on the procedure, but "natural" clusters are sought. Judgments are made as to where the boundaries of these natural clusters occur. Having made judgments on group boundaries, the task of allocating students to them is routine and can be expected to be accurately done in the manual mode. It may, however, be tedious and time consuming considering that the data is most easily analyzed in rank order. The example chosen considers only one data set (pre-test results) on the first sorting into groups. Use of data from more than one test, even if this multivariate data is summarized, is more complex and time consuming in its analysis. Similar considerations apply to the sorting at other levels.

Prior to the assignment of groups to instructional areas must be the assignment of teachers to groups. As a consequence of this constraint, the final number of groups to be used must be deliberately chosen by the instructional staff to fit in with teacher availability and teacher competencies to instruct groups with different needs. In the grouping procedure described here, this recombining of groups or joint instruction of groups by the one teacher is a decision of the instructional staff and is based on a knowledge of previously identified homogeneous groups.

The preceding example involved the grouping of students for instruction in a particular science topic on vertebrates. Such a topic may be one of a set of topics which make up a complete instructional program in science. Although not specified in the earlier illustrative example, topics may be further considered as relatively short-term aggregations of instructional objectives. Within the IGE instructional programming model, instructional objectives are the most specific outcome oriented statements for goal attainment and state for each student what is to be accomplished, at what level of expertise and sometimes by when it will be done (Spuck, Hunter, Owen, and Belt, 1975, page 7).

It should be noted that some instructional programs define prerequisites at either the topic level or the instructional objective level. Instructional objectives or topics within an instructional program may be interrelated in predetermined ways, establishing for the program a network of prerequisites. If such prerequisites exist within a program the objectives are sequential. For example, the achievement of objectives in a mathematics program is often sequential in nature, with completion of lower-order objectives being prerequisite for progress toward higher-order objectives. Not all objectives need be related sequentially, however. Many may be relatively independent, and can be attained at any one of several points in the program of individualized learning. Some instructional programs are characterized by the absence of prerequisites and are therefore non-sequential in nature. The earlier illustrative example on vertebrates may be considered as one topic in a sequence of science topics all of which may be interrelated in a network of prerequisites at the topic level.

Developing Mathematical Processes (DMP) and the Wisconsin Design for Reading Skill Development (WDRSD), both instructional programs developed at the Wisconsin Research and Development Center, contain networks of prerequisites at the instructional objective level. Networks of prerequisites are best described by reproducing Spuck, Hunter, Owen and Belt's diagrammatic illustration (1975, page 25).



(a) Network (b) Linear (c) Non-sequenced

Figure 1-1: Sequencing of Objectives

In a network an objective may have more than a single objective as a prerequisite. For example, objective six in Figure 1a has as prerequisite objectives two and four; objective six is in turn prerequisite to objective seven along with objective five. The linear form (Figure 1b) is clearly a special case of the network form. Figure 1c shows the non-sequenced case. Here, no objective is dependent upon, or prerequisite to, any other objective.

The Wisconsin System for Instructional Management (WIS-SIM), the computer support system which supports IGE, has been designed to assist in the management of instructional programs which contain a prerequisite structure. Belt (1975, page 6) reported that within WIS-SIM the establishment of instructional groups is generally accomplished in two steps.

Firstly, the unit leader or another teacher assesses the overall instructional needs of the students in the unit by examining "Unit Performance Profiles" (see Appendix F, page 375) for the various subject areas under consideration. The Unit Performance Profile summarizes for each student in the unit his past performance in that subject area. Secondly, having assessed the overall status of the students in the unit, a number of instructional groupings are requested from the computer. Each instructional grouping recommendation (IGR) consists of three parts. For each instructional group (skill or topic) requested, there is an Instructional Grouping Recommendation (Group) Report (see Appendix F, page 378) which, in addition to listing the students who are eligible, also indicates any previous experience the student may have had with the instructional objectives. The second part of the IGR is the Instructional Grouping Recommendation (Summary) Report which identifies students eligible for the skills requested. The third part of the IGR is the Instructional Grouping Recommendation (Omissions) Report which lists students who did not qualify for any of the requested instructional groupings. Students fail to qualify either because they have not mastered the necessary prerequisites or because they have already mastered the topic. These grouping recommendations are considered at a meeting of the teachers of the unit who evaluate the grouping recommendations.

In addition to the three-part grouping report, reports are available to the unit leaders in order to establish instructional groups to meet the needs of students not included in the IGR. The Topic Deficiency Report (Appendix F, page 381) lists the specific prerequisite deficiencies which prevented individual students from qualifying for placement in a particular instructional group. The WIS-SIM reports described above illustrate the significant emphasis that IGE places on the establishment of appropriate instructional groups to meet student needs.

From this examination of both IGE and WIS-SIM grouping practices as they were currently being used in some schools, it appeared that an automated grouping procedure should consider the organizational structure of the instructional program being used. It therefore appeared that a grouping procedure, to be useful in hierarchically sequenced instructional programs, should result in groups composed of students all of whom have met the instructional prerequisites set for the topic to be studied. For programs in which the prerequisite structure is not of primary importance, this feature need not be provided. This recommendation is expressed in Criterion IV:

Criterion IV: The grouping procedure should take into account the prerequisite structure of the instructional program when such prerequisites help determine the composition of the groups to be formed.

Factors On Which To Form Groups

The basic data used in a grouping procedure is a set of students on which we have recorded measurements. The initial choice of the particular set of measurements used to describe each student constitutes a frame of reference within which to establish the groupings. The choice reflects the instructional staff's judgment of relevance for the purpose of the grouping and the first question to ask when grouping students concerns the variables and whether the correct ones have been chosen in the sense that they are relevant to the purpose of the grouping. For example, when grouping students for purposes of reading instruction, it is generally not sensible to include such variables as height, weight, and other vital statistics since, the acquisition of reading skills is not considered to be dependent upon these variables. It is important to bear in mind that the initial choice of variables is itself a categorization of the data which has no mathematical or statistical guidelines, and which reflects the investigator's judgment of relevance for the purpose of the grouping.

Everitt (1974, page 12) noted that researchers in the natural sciences take a unique approach to this problem through the "hypothesis of nonspecificity." Briefly, it is assumed that the grouping structure (considered as unconstrained) is dependent on many variables, any single one of which can be deleted or added without noticeable effect. As a consequence, numerical taxonomy or clustering investigations often involve huge numbers of variables. On the other hand, behavioral and social scientists, statisticians, and engineers strongly emphasize parisomy and seek to minimize the number of measured variables. This approach puts a premium on wise selection of variables both for relevance and discriminating power.

Lockhart and Liston (1970, page 8), when commenting on the clustering of data in microbiology, cautioned about types of variables on which not to base the formation of groups. Their advice, although directed towards a non-educational field, is pertinent in considerations of relevance and discrimination.

1. Meaningless characteristics which do not have proven direct or indirect affect on the purpose for the grouping should not be considered.

2. Characteristics positive, negative or very much of the same magnitude for all units in the original group should not be included in the list of factors on which to form groups as they do not provide useful discriminatory information.

3. Redundant characteristics should be avoided. Inclusion of a series of obviously and closely linked or correlated characteristics will not provide very much extra discriminatory power in the formation of groups.

4. It is mandatory that precise methods and exact definitions be employed in testing. Vague terms are inappropriate for identification purposes and inappropriate for computer coding. Presence/absence, positive/negative, character states representing exact categories for quantitative character rankings on variables or variables capable of interval or ratio measurement are best for coding.

The selection of factors on which to form instructional groups should be based on these considerations of parsimony, relevance, and discrimination. Already the review of grouping procedures in the previous section has led to the identification of several relevant factors or sets of factors. These include achievement scores, learning styles and interest levels as recommended in the IGE literature (Sorenson, Poole, and Joyal, 1976, page 11). Although it cannot be claimed that the use for grouping purposes of these factors is widespread, nevertheless, a grouping procedure should make provision for their use. Particularly, consideration needs to be given to their measurement and the effects of compounding their measures into indices of similarity.

According to their purposes, teachers may choose to group children by a number of different criteria such as: aptitudes, achievement, interest, sociometric choice, and the task at hand (Sorenson, Poole and Joyal, 1976, page 12).

Aptitudes

Aptitude is potential ability for achievement. Abilities in the sense of special aptitudes are frequently criteria for grouping. Students with comparable aptitudes for a foreign language, for example, may be placed in the same language groups. Students with comparable physical abilities may similarly be grouped into teams for sports activities.

The most commonly used criterion involving ability, however, is probably that of mental ability. Mental ability as measured by scores on standardized tests is rarely used as a sole criterion for grouping. In most classes where teachers consider mental ability test scores as a basis for grouping, achievement scores are also taken into account. Social studies groups, for instance, may be based on mental ability and reading achievement as well as past grades in social studies.

Achievement

Achievement tests measure the present proficiency, mastery, and understanding of general and specific areas of knowledge. Achievement tests are either standardized or specially constructed tests. Standardized achievement tests can also be classified into general and special tests. General tests are typically batteries of tests that measure the most important areas of school achievement: language usage, vocabulary, reading, arithmetic, and social studies. Special achievement tests are tests in individual subjects such as history, science, or English.

Perhaps reading achievement data has been most frequently used as the basis for classroom grouping. It has been used not only to determine reading groups, but also for groups in social studies and other subjects.

Achievement data from the curriculum area in which the groups are to be formed is commonly utilized, and in IGE a further breakdown within the curriculum area according to the instructional objectives is frequently necessary. For example, a student might have high achievement scores in mathematics in sets and geometry, but medium to low scores in computational areas.

Interests

As a criterion for grouping, interest takes into account an important dimension of learning that grouping by ability and achievement may neglect—namely, motivation. If a child is to be motivated to use his ability, he must be interested in the task at hand. Many teachers, understanding the role that motivation plays in learning, group for interest. Students interested in learning about vertebrates, for example, will be more likely to work effectively if grouped together than if forced to choose some other science topic in which they have little interest. Interest inventories useful in measuring interest in different areas employ criterion keying of items. The assumption is that the subject's responses to a set of items, presumably representative of particular vocations or areas, indicates his interest level in that area. His responses are then compared to the responses of members of the various vocations or areas. The Kuder Preference Record (Anastasi, 1961, page 538) yields a profile of responses measured as percentiles along scales each representing a particular vocation or area.

The measurement of the attitudes and values of students for purposes of forming instructional groups does not seem to be recommended in the literature. However, objective tests are available for these purposes and most often result in ordinal level measurements. Their use for grouping purposes does not seem to pose any unique difficulties.

Sociometric Choices

This is another basis for grouping. By informal means and by using sociograms, teachers are able to analyze the patterns of social interaction in their classes. Then can identify the students who are at the center of clique groups, the students who make mutual choices, and those who are isolated. They are aware of the roles different children play in different classroom groups.

Teachers can use their sociometric knowledge to help place apparently isolated and rejected children in group situations where they are most likely to interact effectively with others.

Learning Style

Learning style is variously defined in terms of factors which aid the learning of individual students. For example, Bechtal (1971, page 46) defined learning style in individualized instruction programs as "those factors that ease and facilitate learning for an individual student in a given situation. Dunn and Dunn (1972, page 29) provided the following twelve elements of learning style as identified by use of an observation schedule with each element measured on a Likert scale.

1. Time most alert
2. Attention span
3. Tolerable noise level
4. Type of sound
5. Type of work group
6. Amount of pressure
7. Type of pressure and motivation
8. Place
9. Physical environment and conditions
10. Type of assignments
11. Perceptual strengths and styles
12. Type of structure and evaluation

Hunt and Sullivan (1973, page 221) proposed three categories of learning style based on a conceptual levels dimension that ranges from a very concrete level at which the person is unsocialized and capable of only very simple information processing to a complex stage where the person is self responsible and capable of processing and organizing information in a complex fashion. Hunt and Sullivan utilized a written response test which they use to categorize the learning styles of respondents.

The Witchita Public Schools System in Kansas (1975, page 1) is developing an instrument to measure learning style in three broad categories of (a) information-gathering/receiving, (b) social work conditions, and (c) expressiveness preference. Within each of these areas are subtopics as follows:

(a) The area of information gathering/receiving considers:

1. Auditory Language: The way a student hears words; processing spoken words.
2. Visual Language: The way a student sees words; processing written language.
3. Auditory Numerical: The way a student hears numbers; processing spoken numerical values.
4. Visual Numerical: The way a student sees numbers; processing written numerical values.
5. Auditory-Visual-Kinesthetic: The way a student learns by doing or involvement. Emphasizing the experiencing or manipulative learning style which is almost always accompanied by either auditory stimuli, visual stimuli, or a combination of both.

(b) The area of working considers whether a student likes to work or learn in a group or alone. They are appropriately titled as follows:

1. Group learner: A student who likes to work with at least one other person when there is important work to be done.
2. Individual learner: A student who works and thinks best alone. This student is usually a self-starter and frequently finds working with other students distracting.

(c) The area of individual expressiveness considers how a student prefers to express himself. Basically, they fall into one of these two broad categories:

1. Oral expressive: A student who prefers to say what he knows. Usually, answers or explanations are better given orally; however, some students may indicate this preference simply because they are too lazy to write things down.
2. Written expressive: A student who prefers to write down answers or explanations. Students who exhibit a reflective cognitive learning style may prefer this method.

The Kansas instrument, currently being developed, requires the administration of a twenty-minute objective test. The administration can be given to a group and results in scores on each of the nine dimensions. Scores 33 through 40 are considered as indicating a major learning style, scores from 20 through 32 as a minor learning style and below 20 indicates the student uses this style to a negligible extent.

Measurement of Factors Used in Grouping

Ability, aptitude, achievement, interest and learning style test scores are ordinal. They indicate the rank order positions of the students. The scales used in their testing neither have equal intervals or absolute zeros. Therefore, strictly speaking the statistics (e.g., similarity indices) that can be used with ordinal scales do not include statistics such as r, t, or F. However, as Kerlinger (1973, page 440) noted in the measurement of preferences and attitudes, for example, the neutral points of a Likert type scale can be considered natural origins. Furthermore, Kerlinger (page 440) adopted a pragmatic viewpoint about making the assumption of equal intervals measurement for data which are strictly speaking ordinal. He opines that the assumption works. It is probable that most psychological and educational scales approximate interval equality fairly well. However, in making this assumption, care must be taken in (1) noting scales which possess gross inequality of scale and (2) in the interpretation of the obtained measurements.

From the foregoing description of relevant variables and their measurements, the data set used in grouping students can be expected to be heterogeneous; that is, of the same type but of different scales (Hartigan, 1975, page 50). Although it appears likely that the most commonly occurring scale will be ordinal (perhaps considered as an interval scale), it is possible that a categorical scale may be required in the measurement of some variables. This likelihood found expression in the following criterion of acceptability for a grouping procedure.

Criterion V. The grouping procedure should permit the selection and use of data measured on different scales as this is considered relevant to the purpose of grouping.

A brief summary of quantitative models for grouping used in some non-educational areas follows. These clustering techniques, already introduced in section 1-1, will be examined for their purposes and characteristics in an attempt to ascertain their relevance to the problem of grouping students for instructional purposes. This survey will be preliminary to a detailed examination of those techniques considered as being most relevant in the solution of the problem.

Numerical Grouping Procedures

Certain numerical grouping procedures which had been found useful in other areas of study were considered to be potentially useful for instructional purposes. These grouping procedures can be based on student characteristics and produce sets of possible groupings. Some procedures take into account constraints such as numbers of groups. "Cluster analysis" is the generic term for these techniques which are useful in the analysis of multivariate data and which result in the grouping of similar objects. Some of these clustering techniques attempt to solve the problem: Given n objects or individuals, each of which is measured on each of p variables, devise a classification scheme for grouping the objects into g classes such that the similarity between pairs of objects in the same group is greater than between pairs of objects in different groups.

The need for cluster analysis has arisen in a natural way in many fields of study--the life sciences (e.g., botony), the behavioral and social sciences (e.g., psychology), the earth sciences (e.g., geology), medicine (e.g., psychiatry), engineering sciences (e.g., pattern recognition) and the information and policy sciences (e.g., information retrieval). However, it is only since computers, which can take the burden of the very large amounts of computation generally involved have become available, has much attention been given to clustering procedures. Consequently, this field of study is as yet relatively undeveloped, and mathematical statisticians only recently have begun to formalize clustering procedures of which there are numerous examples (Anderberg, 1973; Everitt, 1974, and Hartigan, 1975). Since the use of clustering procedures in education had been infrequent (Baker, 1972, page 1 and McRae, 1971(b), page 3), little was known regarding their utility in forming groups for instructional purposes; however, given the importance of grouping students for instructional purposes, the examination, selective application, and evaluation of clustering procedures seems warranted.

Clustering techniques have been classified into types by Everitt (1974, page 7) as follows:

1. Often a hierarchy of clusters is sought, rather than one level of clustering. In the hierarchical techniques, the classes (clusters) are themselves classified into groups, the process being repeated at different levels to form a tree or family of clusters. The tree may be represented diagrammatically as a dendrogram.

2. Optimization-partitioning techniques which depend upon establishing clustering centers and which then grow in size by merging other objects into the clusters. The central idea in most of these methods is to choose some initial partition of the objects and then alter cluster memberships so as to obtain a better partition.

3. Density or mode-seeking techniques attempt to compare relative distance between entities (considered as points in metric space) and to search for continuous relatively densely populated regions of the space surrounded by continuous relatively empty regions.

4. Clumping techniques, unlike most classification techniques, permit an overlap between the clusters. The clusters produced by these techniques are not mutually exclusive; that is, an entity may be a member of more than one group or cluster.

5. Other methods which do not fall clearly into any of the four previous groups; for example, factor analysis and discriminant function analysis.

These types of clustering techniques are now briefly described. In the descriptions, "entities" are the individuals or objects which are to be placed into groups or clusters.

Hierarchical Clustering Techniques

Hierarchical techniques may be subdivided into agglomerative methods which proceed by a series of successive fusions of the N entities into groups and divisive methods which partition the set of N entities successively into finer partitions. The results of both agglomerative and divisive techniques may be presented in the form of a dendrogram, which is a two-dimensional diagram illustrating the fusions or partitions which have been made at each successive level.

Agglomerative Methods

The basic procedure with all these methods is similar. They begin with the computation of a similarity or distance matrix between the entities. For example, a very common similarity coefficient is the product moment correlation coefficient, and perhaps the most common distance measure is Euclidean distance.

At any particular stage the methods fuse individuals or groups of individuals which are closest (or most similar). Differences between methods arise because of the different ways of defining distance (or similarity) between an individual and a group containing several individuals, or between two groups of individuals. Some of the methods are only really suitable for use when a distance matrix is used as the starting point, and where this is so it will be noted. Several agglomerative hierarchical techniques are now described and for convenience the description will be in terms of distance measures.

(i) The Nearest Neighbor or Single Link Method

This method can be used both with similarity measures and with distance measures. Groups initially consisting of single individuals are fused according to the distance between their nearest members, the groups with the smallest distance being fused. Each fusion decreases by one the number of groups. For this method, then, the distance between groups is defined as the distance between their closest members. Everitt (1974, page 9) provided the following example in which five individuals are to be classified, and the matrix of distances between the individuals, namely D_1, is as follows:

$$D_1 = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0.0 & 2.0 & 6.0 & 10.0 & 9.0 \\ 2 & 2.0 & 0.0 & 5.0 & 9.0 & 8.0 \\ 3 & 6.0 & 5.0 & 0.0 & 4.0 & 5.0 \\ 4 & 10.0 & 9.0 & 4.0 & 0.0 & 3.0 \\ 5 & 9.0 & 8.0 & 5.0 & 3.0 & 0.0 \end{array}$$

(In this matrix the element in the ith row and jth column gives the distance, d_{ij}, between individuals i and j.)

At stage one of the procedure individuals 1 and 2 are fused to form a group, since d_{12} is the smallest entry in the matrix D_1. The distance between this group and the three remaining single individuals 3, 4, and 5, are obtained from D_1 as follows:

$$d_{(12)3} = \min [d_{13}, d_{23}] = d_{23} = 5.0,$$

$$d_{(12)4} = \min [d_{14}, d_{24}] = d_{24} = 9.0,$$

$$d_{(12)5} = \min [d_{15}, d_{25}] = d_{25} = 8.0$$

A new distance matrix D_2 giving inter-individual distances, and group-individual distances may now be formed.

$$D_2 = \begin{array}{c|cccc} & (12) & 3 & 4 & 5 \\ \hline (12) & 0.0 & 5.0 & 9.0 & 8.0 \\ 3 & 5.0 & 0.0 & 4.0 & 5.0 \\ 4 & 9.0 & 4.0 & 0.0 & 3.0 \\ 5 & 8.0 & 5.0 & 3.0 & 0.0 \end{array}$$

The smallest entry in D_2 is d_{45} which is 3.0, and so individuals 4 and 5 are fused to become a second group, and distances now become

$$d_{(12)3} = 5.0 \quad \text{(as before)}$$

$$d_{(12)(45)} = \min\,[d_{14}, d_{15}, d_{24}, d_{25}] = d_{25} = 8.0,$$

$$d_{(45)3} = \min\,[d_{34}, d_{35}] = d_{34} = 4.0$$

These may be arranged in a matrix D_3,

$$D_3 = \begin{array}{c|ccc} & (12) & 3 & (45) \\ \hline (12) & 0.0 & 5.0 & 8.0 \\ 3 & 5.0 & 0.0 & 4.0 \\ (45) & 8.0 & 4.0 & 0.0 \end{array}$$

The smallest entry now is $d_{(45)3}$ and so individual 3 is added to the group containing individuals 4 and 5. Finally fusion of the two groups at this stage takes place to form a single group containing all five individuals. The dendrogram showing these fusions appears below:

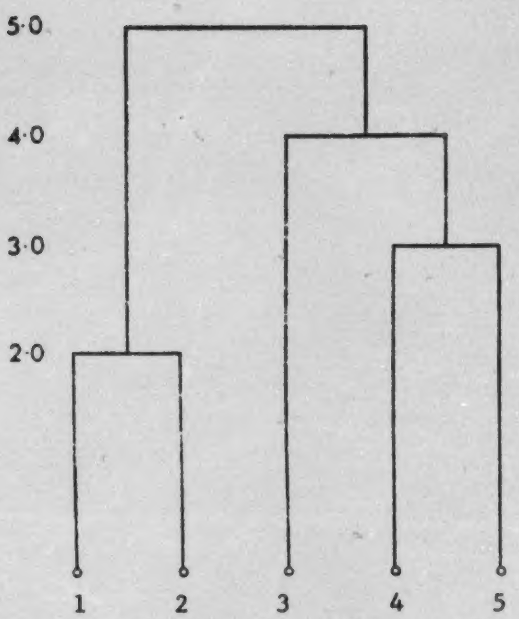


Figure 1-2: Dendrogram Using Single Link Method

This technique seems to have been first described by Sneath (1957), and later by Johnson (1967).

(ii) The Furthest Neighbor or Complete Linkage Method

This method is exactly the opposite of the single linkage method, in that distance between groups is now defined as the distance between their most remote pair of individuals. This method can also be used with similarity and distance measures.

(iii) Centroid Method

In this method, groups are depicted to lie in Euclidean space, and are replaced on formation by the coordinates of their centroid. The distance between groups is defined as the distance between the group centroids. The procedure then is to fuse groups according to the distance between their centroids, the groups with the smallest distance being fused first. Again, this method can be used with both similarity and distance measures.

(iv) Group Average Method

This method defines distance between groups as the average of the distances between all pairs of individuals in the two groups. Sokal and Michener (1958) used this average as a measure of distance between an individual and a group of individuals, while Lance and Williams (1966, page 60) extended it to a measure of distance between groups.

The procedure can be used with similarity and distance measures provided the concept of an average measure is acceptable.

(v) Ward's Method

Ward (1963, page 236) proposed that at any stage of an analysis the loss of information which results from the grouping of individuals into clusters can be measured by the total sum of squared deviations of every point from the mean of the cluster to which it belongs. At each step in the analysis, union of every possible pair of clusters is considered and the two clusters whose fusion results in the minimum increase in the error sum of squares are combined. Everitt (1974, page 15) provided the following example in which five individuals are to be clustered on the basis of their values on a single variable using this method of cluster analysis. The values of the variable for each of the five individuals are:

		Variable Value
Individual	1.	1
	2.	2
	3.	7
	4.	9
	5.	12

The error sum of squares (E.S.S.) is given by:

$$\text{E.S.S.} = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\Sigma x_i)^2$$

where x_i is the score of the ith individual. At stage one, each individual is regarded as a single member group and so E.S.S. is zero. The two individuals whose fusion results in the minimum increase in E.S.S. form the first group, and for our data these are individuals 1 and 2 and the E.S.S. becomes 0.5. At the next stage individuals 3 and 4 fuse to form a second group, increasing the E.S.S. by 2.0 to 2.5. Next, individual 5 joins the group formed by 3 and 4, and the E.S.S. increases by 12.7 to 15.2. Finally, the two remaining groups are fused and the E.S.S. increases by 71.6 to 86.8.

These results may be summarized as a dendrogram which is shown in Figure 1-3.

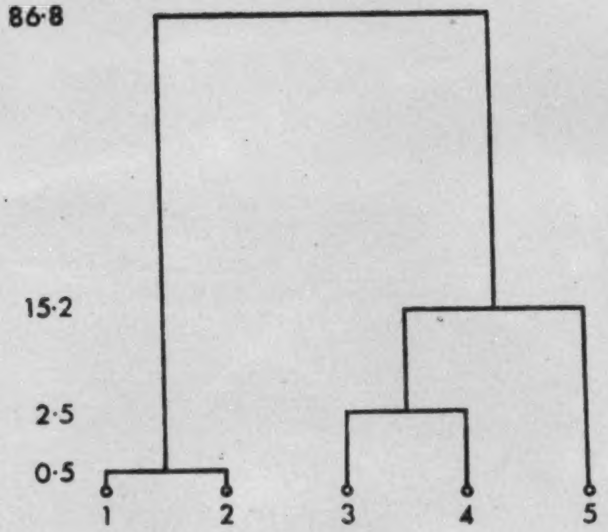


Figure 1-3: Dendrogram for Ward's Method

Probably the majority of the applications of agglomerative hierarchical techniques have been in the fields of biology and zoology. They are particularly useful for plants and animals when these are hierarchically grouped with respect to genetic characteristics.

Baker (1972, page 352) described the application of hierarchical procedures to an instructional grouping situation and noted their usefulness in providing information about the dynamics of group formation.

Divisive Methods

With divisive methods the first task is to split the initial set of individuals into two. Now a set of n individuals can be divided into two subsets in $2^{n-1} - 1$ ways, and although Edwards and

Cavalli-Sforza (1965, page 362) considered them all, this is obviously only possible for very small sets, even with a large computer. In the case of even moderately large sets we have to impose a restriction on the number of ways considered. There are two types of divisive techniques: monothetic, which are based on the possession or otherwise of a single specified attribute, and polythetic, which are methods based on the values taken by all the attributes.

Everitt (1974, page 18) considered the most feasible of the polythetic divisive techniques is that described by MacNaughton-Smith et al. (1964, page 1034). In this instance a splinter group is accumulated by sequential addition of the entity whose total dissimilarity with the remainder less its total dissimilarity with the splinter group is a maximum. When this difference becomes negative the process is repeated on the two subgroups. The measure of dissimilarity used is the average Euclidean distance between each entity and the other entities in the group. For example, the distance matrix D, shown below, gives the distances between seven individuals.

$$D = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 0 & 10 & 7 & 30 & 29 & 38 & 42 \\ 2 & 10 & 0 & 7 & 23 & 25 & 34 & 36 \\ 3 & 7 & 7 & 0 & 21 & 22 & 31 & 36 \\ 4 & 30 & 23 & 21 & 0 & 7 & 10 & 13 \\ 5 & 29 & 25 & 22 & 7 & 0 & 11 & 17 \\ 6 & 38 & 34 & 31 & 10 & 11 & 0 & 9 \\ 7 & 42 & 36 & 36 & 13 & 17 & 9 & 0 \end{array}$$

These individuals are to be divided into two groups using this method. The individual used to initiate the splinter group is the one whose average distance from the remaining individuals is a maximum. This is found to be individual 1, giving the groups

(1) and (2, 3, 4, 5, 6, 7).

Next, the average distance of each individual in the main group to the individuals in the splinter group is found, followed by the average distance of each individual in the main group to the other individuals in this group. The difference between these two averages is then found. This gives the following results:

Individual	Average Distance to Splinter Group (1)	Average Distance to Main Group (2)	(2-1)
2	10.0	25.0	15.0
3	7.0	23.4	16.4
4	30.0	14.8	-15.2
5	29.0	16.4	-12.6
6	38.0	19.0	-19.0
7	42.0	22.2	-19.8

The maximum difference is 16.4 for individual 3, who is therefore accumulated into the splinter group giving the two groups:

(1,3) and (2, 4, 5, 6, 7).

This method has the advantage that the computation required is considerably less than for an all possible subdivisions method. As with other divisive techniques, an inefficient early partition cannot be corrected at a later stage. This is also the case for agglomerative techniques.

Monothetic techniques are usually used in cases where binary data is used. A division of the set of data is then initially into those individuals who possess, and those who lack, some one specified attribute. If only divisions of this simple type are considered then, given data for, say, m attributes, there are m potential divisions of the initial set, (m-1) potential divisions of each of the two subsets thus formed, and so on. Such a division is termed monothetic, and a hierarchy of such divisions a monothetic classification. Association analysis (Lance and Williams, 1965, page 246) is a monothetic technique.

Partitioning Techniques

This section describes clustering techniques which produce a partition of the objects, but differ from the hierarchical techniques in that they admit relocation of the entities, thus allowing poor initial partitions to be corrected at a later stage.

The majority of these techniques can be formulated as attempts to partition the set of entities so as to optimize some predefined criterion. For example, many of them attempt to minimize trace (W), where W is the pooled within groups matrix of sums of squares and cross products. Most of the methods also assume that the number of groups has been decided a priori by the investigator, although some do allow the number to be changed during the course of the analysis. Most of these techniques employ three distinct procedures, which are as follows:

(a) A method of initiating clusters;

(b) A method for allocating entities to initiated clusters;

(c) A method of reallocating some or all of the entities to other clusters once the initial classificatory process has been completed.

The differences between the methods lie primarily in (a) and (c).

Techniques Used for Initiating Clusters

The majority of techniques begin by finding k points in the p-dimensional space, which act as initial estimates of the cluster centres (where k equals the number of groups to be formed). Various procedures have been suggested for choosing these points which are known as seed points. For example, MacQueen (1967, page 285) chose the first k points in the sample as the initial k cluster mean vectors. Anderberg (1973, page 157) mentioned seven other methods of establishing seed points, including random selection, sequential selection, subjective selection, selection of the centroids of any desired initial partition and the selection of seed points which are greater than some specified distance from each other.

The k starting points are used as initial estimates of cluster centres. Entities are allocated to the cluster to whose centre they are nearest (usually in the Euclidean metric), and the estimate of the centre may be updated after the addition of each entity to the cluster (MacQueen, 1967, page 287), or only after all the entities have been allocated (Ball and Hall, 1967, page 153).

It is often useful if the investigator, on the basis of prior knowledge, can specify the initial cluster configuration.

Relocation Techniques

Once an initial classification has been found by one of the methods mentioned above, a search is made for entities which should be reallocated to another group. This relocation takes place in an attempt to optimize some clustering criterion.

In general relocation proceeds by considering each entity in turn for reassignment to another cluster, reassignment taking place if it causes an increase (or decrease in the case of minimization) in the criterion value. The procedure is continued until no further move of a single entity causes an improvement. A local optimum of the criterion value is thus reached and the solution may be accepted or an attempt may be made to improve it by repeating the procedure using a different starting configuration. In general there is no way of knowing whether or not the absolute maximum of the criterion has been reached.

Clustering Criterion

Three illustrative clustering criterion are all derived from the following fundamental matrix equation:

$$T = W + B$$

where T is the total scatter or dispersion matrix, W is the matrix of within -groups dispersion—that is, $W = \sum_{i=1}^{g} W_i$ where W_i is the dispersion matrix for group i—and B is the between -groups dispersion matrix.

For any given data set the matrix T is fixed, and so functions of B and W are sought as clustering criteria.

(i) Trace (W)

The first criterion is the minimization of the trace of the pooled-within groups matrix of sums of squares and cross products. It has been proposed alike by Friedman and Rubin (1967), MacRae (1971), and Calinski and Harabasz (1971): It may be shown that this criterion is the same as minimizing the total within group sum of squares of the partition (Everitt, 1974, page 26).

(ii) Determinant of W

The next criterion is the minimization of the determinant of the within-cluster matrix of sums of squares and cross products. This criterion seems to have been first suggested as a clustering criterion by Friedman and Rubin (1967).

(iii) Trace BW^{-1}

Another criterion suggested by Friedman and Rubin (1967) is the maximization of the trace of the matrix BW^{-1}, obtained from the product of the between-groups matrix of sums of squares and cross products and the inverse of the within-groups matrix.

The description of partitioning techniques will not include specific procedures at this stage. A detailed discussion of the elements, processes, and logic of partitioning procedures will be provided in Chapter II together with an analytical comparison of different partitioning techniques.

Density Search Techniques

If entities are depicted as points in a metric space, a natural concept of clustering suggests that there should be parts of the space in which the points are very dense, separated by parts of low density. This concept was used by Gengerelli (1963), and Charmichael et al. (1968). Methods of cluster analysis which use this approach of seeking regions of high density or modes in the data are known as density search techniques. In general each mode is taken to signify a different group.

Several of these methods have their origins in single linkage cluster analysis. One example of a density search technique is now described.

The Taxmap Method of Carmichael and Sneath

The density seeking technique considered here is one due to Carmichael et al. (1968), and later extended by Carmichael and Sneath (1969). It attempts to imitate the procedure used by the human observer for detecting clusters in two or three dimensions, that is to compare relative distance between points, and to search for continuous relatively densely populated regions of the space surrounded by continuous relatively empty regions. Clusters are formed initially in a way similar to that previously described for the single linkage method, but criteria are adopted for judging when additions to clusters should be stopped. One such criterion is to terminate additions if the prospective point is much further away than was the last point admitted, as indicated by a discontinuity in closeness.

For this purpose the authors use a single measure obtained by subtracting the drop in the average similarity on addition of an entity to the cluster, from the new average. This measure has been found to decrease smoothly until there is a discontinuity whereas the drop in average similarity by itself may vary widely. Everitt (1974, page 31) provides the following matrix of similarities between five individuals:

$$S = \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ 1 & 1.0 & 0.7 & 0.9 & 0.4 & 0.3 \\ 2 & 0.7 & 1.0 & 0.8 & 0.5 & 0.4 \\ 3 & 0.9 & 0.8 & 1.0 & 0.4 & 0.2 \\ 4 & 0.4 & 0.5 & 0.4 & 1.0 & 0.7 \\ 5 & 0.3 & 0.4 & 0.2 & 0.7 & 1.0 \end{array}$$

The two most similar individuals are used to initiate the cluster. From S these are found to be individuals 1 and 3, whose similarity is 0.9. The next point considered for admission to the cluster is that one most similar to a point already in the cluster. This leads to consideration of individual number 2 whose similarity with individual 3 is 0.8. The average similarity between the three individuals is now computed to give the following:

Cluster members	Candidate individual
1,3	2
Similarity 0.9	

Average similarity between individuals 1, 3 and 2 is

$$\frac{1}{3}(0.9+0.7+0.8) = 0.8.$$

Therefore the drop in similarity is 0.9-0.8=0.1, and hence the measure of discontinuity is (0.8-0.1)=0.7. Low values of this measure indicate that the candidate point should not be added to the cluster. If in this case "low values" were regarded as those less than 0.5, then individual 2 would be added to the cluster, and a further individual would be considered for admission as follows:

Cluster members	Candidate individual
1, 3, 2	4

Similarity 0.8

Average similarity between individual 1, 3, 2 and 4 is

$$\frac{1}{6}(0.9+0.7+0.4+0.8+0.5+0.4) = 0.6.$$

Therefore the drop in similarity is 0.2, and the measure of discontinuity is 0.4. This individual is therefore not admitted to the cluster, but initiates a new one.

Various other criteria are also used to prevent admission of points relatively near the centroid of an elongated cluster but still rather far from any point in it.

Clumping Techniques

Most classification techniques lead to distinct or disjoint clusters, and these are what is required in most fields of application. In other cases (language studies, for example) classification must permit an overlap between the classes if it is to be of any value, because words tend to have several meanings, and if they are being classified by their meanings they may belong in several places. In general, classification techniques which allow overlapping clusters are known as clumping techniques.

Clumping techniques begin with the computation of a similarity matrix from the original data to give an estimate of the similarity between each pair of entities on the basis of the properties they exhibit. These methods seek a partition of the entities into two groups, the smaller of which is generally considered to be the class sought. Partitions are found by minimizing a cohesion function between the two groups. For example, Needham (1967, page 48) considered a symmetric cohesion function G(A) given by

$$G(A) = \frac{S_{AB}}{S_{AA}S_{BB}}$$

where A and B refer to the two groups into which the data are partitioned, A being the putative clump. $S_{AB} = \sum_{i \in x} \sum_{j \in y} S_{ij}$, where S_{ij} is an inter-entity similarity coefficient. Algorithms to minimize these functions proceed by successive reallocations of single individuals from an initial randomly chosen cluster centre. By iterating from different starting points many partitions into two groups may be found. In each case the members of the smaller group are noted and constitute a class to be set aside for further examination. This independent search for classes is the reason for one of the less attractive features of these methods, namely that it is not at all unusual for the same class to be found several times; no way is known of completely avoiding this.

Other Clustering Techniques

The methods described in the preceding sections constitute some of the most recent work in the field of cluster analysis. There remain, however, several other clustering techniques which have been found useful, and which do not fall clearly into any of the four previous categories. Some of these techniques will now be described.

The usual starting point for a cluster analysis is an n x p data matrix in which the scores of n individuals (or objects) for p variables are recorded. In factor analysis, the matrix of interest is a p x p similarity matrix where p is the number of variables. The purpose of the analysis is to find a n x r matrix F, where $r<p$, such that FF' reasonably approximates the original similarity matrix. The factor analysis model was developed as a partitioning of variance into linear components. In this analysis, the variables are taken as a fixed, complete representation of the domain of interest and the individuals are taken as a random sample, the analysis transforms the variables into linear components.

As an approach to the clustering or grouping of individuals, it has been suggested that one might factor analyze an n x n matrix of similarities among the n individuals (Stephenson, 1953). In this analysis, called Q-factor analysis, the n individuals are assigned to clusters on the basis of their scores on the n factors. The individuals, not the variables, should be taken as a fixed, complete representation of the domain to be partitioned. However, this is not generally the intention of the researcher employing Q-techniques who would like to consider his data as coming from a random sample of individuals and would like to be able to generalize to the population of individuals of interest (McRae, 1973, page 4).

Discriminant analysis attempts to answer the question: How can individuals best be assigned to already existing groups on the basis of several variables? Kerlinger (1973, page 650) described a discriminant function as a regression equation with a dependent variable that represents group membership. The function maximally discriminates the members of the group; it indicates to which group each new member probably belongs.

From the above descriptions of available clustering techniques, it is apparent that some are not directly applicable to the problems of grouping students and that none are exactly applicable to the grouping situation as this is defined by Criteria I through V. For example, none of the techniques reviewed directly refer to eligibility for group membership. Clustering methods are mostly used where naturally occurring clusters are sought. The formation of these clusters is typically free of eligibility constraints. However, the partitioning techniques, which are very similar to the steepest descent algorithms used for unconstrained optimization problems in non-linear programming, appear most directly amenable to such restrictions on eligibility for group membership.

Similarly, none of the applications of the techniques reviewed have incorporated the option of prespecifying the sizes of the groups either as exact sizes or as size ranges. Again it seems that the partitioning techniques are the most directly adaptable for providing this option.

More of the techniques meet the criteria of prespecification of the number of groups to be formed. Q-factor analysis permits the number of groups of persons to be considered, hierarchical techniques (either the agglomerative or diversive types) can be terminated at the desired level and the partitioning techniques are mostly designed on the basis of a fixed number of groups.

All techniques permit the use of multivariate data of the type commonly used in grouping for instructional purposes. Neither does the number of variables likely to be considered in the grouping situation appear to be a restriction of any of the techniques.

Only the clumping techniques do not result in disjoint or mutually exclusive clusters and therefore may be rejected as a procedure for grouping students. Discriminant analysis may also be rejected as a procedure for grouping students because of its assumption of the presence of already existing and clearly defined groups.

The partitioning techniques are the only techniques of those examined which attempt to optimize a clustering criterion. This criterion is often the total within groups sum of squares which can be taken to represent the degree of homogeneity possessed by the groups. On the other hand, most hierarchical grouping procedures focus attention on the dynamic and successive formation of groups and in so doing do not attempt to optimize a criterion. Ward's variance hierarchical method does, however, at each step in the analysis consider the union of every possible pair of clusters and the two clusters whose fusion results in the minimum increase in the within sum of squares are combined.

Hierarchical clustering techniques also have a general disadvantage since they contain no provision for reallocation of entities who may have been poorly classified at an early stage of the procedure. In other words there is no possibility of correcting for a poor initial partition. The partitioning techniques do not share this disadvantage but do, however, possess the disadvantage of mostly resulting in sub-optimal solutions. When complete enumeration of all possible partitions is infeasible, as is mostly the case, an initial partition is provided by the investigator. The optimum value of the criterion is therefore only local to the initial partition. The usual procedure then is to compare the values of the criteria obtained from several other initial partitions and then select the minimum criterion value. With well structured data different starting partitions will usually lead to the same final solution, but in general there is no way of knowing if the criterion value obtained is the true optimum or only a local optimum.

Despite the limitation of providing only local optima, it appears that the partitioning techniques have the most direct application to the grouping of students for instructional purposes. They appear to most closely meet the Criteria I through V and their general structure appears to be adaptable to better meet these criteria. It therefore seems profitable to limit a more detailed examination of clustering techniques to the optimization-partitioning techniques. This is carried out in Chapter II and serves as the basis for the development of an acceptable procedure for grouping students for instructional purposes.

This chapter has been concerned with an examination of the educational climate within which the problem was set and also with an introductory description of some grouping techniques used in other areas of inquiry. This permitted a more definitive statement of the problem.

Statement of the Problem

Approaches to individualization of instruction such as IGE are dependent to a large degree upon the appropriate formation of instructional groups for the extent to which they adequately cater for the educational needs of individual students. This study involved the development of an automated procedure useful in forming groups of students for instructional purposes. This aim finds expression in the following problem statement.

Can a computerized numerical procedure be developed which groups elementary school students for instructional purposes and which takes account of the following factors:

1. Considers a range of skills or objectives for which students may be eligible and places into a group only those students who have met the prerequisites of the objective or skill and who have not mastered that objective or skill.
2. Permits the prior determination by teachers of the number of groups to be formed.
3. Permits the prior determination of the exact size or size range of each of the groups to be formed.

4. Assigns students to groups on the basis of measures of relevant student learning characteristics such as measures of prior achievement and learning style.
5. Assigns students to groups so as to maximize the homogeneity of these groups as measured by the degree of similarity amongst those characteristics.

On the assumption that more than one such mathematical model could be identified considerations for its implementation in the instructional setting led to three related research questions.

1. Which grouping procedure of those compared yields the most homogeneous groupings?
2. Are the groupings formed on the basis of the numerical grouping procedure more homogeneous than teacher created groups?
3. Do teachers involved in the groupings of students perceive the computerized grouping procedure as being a more efficient procedure than those procedures currently employed, as being able to take into account (a) realistic constraints on the formation of groups and (b) relevant learner characteristics? "More efficient" was defined as "less time taken in the grouping process." "Realistic constraints" were those which pertain to personnel availability as these affect the number of groups and their sizes. Relevant learner characteristics includedstudents' prior achievement, rate of learning, and learning style.

Preliminary design considerations for acceptable partitioning procedures are presented in Chapter II. These considerations arose out of an examination of currently available optimization - partitioning techniques.

CHAPTER II

FURTHER DESIGN CONSIDERATIONS

An examination of the operations research literature (Hillier and Lieberman, 1974 and Wagner, 1975) revealed that the sub-optimal partitioning techniques introduced in Chapter I are a subset of a wider collection of optimization procedures designed to solve combinatorial problems. The literature on cluster analysis (Anderberg, 1973 and Everitt, 1974) focused on the sub-optimal techniques and ignored other techniques which yield exact solutions. Because the problem being investigated involved (1) the search for an algorithm directly applicable to the grouping of students for instructional purposes and (2) the possible subsequent modification of an existing algorithm, it appeared appropriate to review the wider collection of combinatorial procedures and their application to assignment problems. The various procedures reviewed are displayed in Figure 2-1. This review resulted in a series of recommendations which formed the basis for the design of an acceptable computerized grouping procedure.

Complete Enumeration

The problem of placing n students, each measured on p variables, into g groups, such that the similarity between pairs of individuals in the same group is greater than between pairs of students in

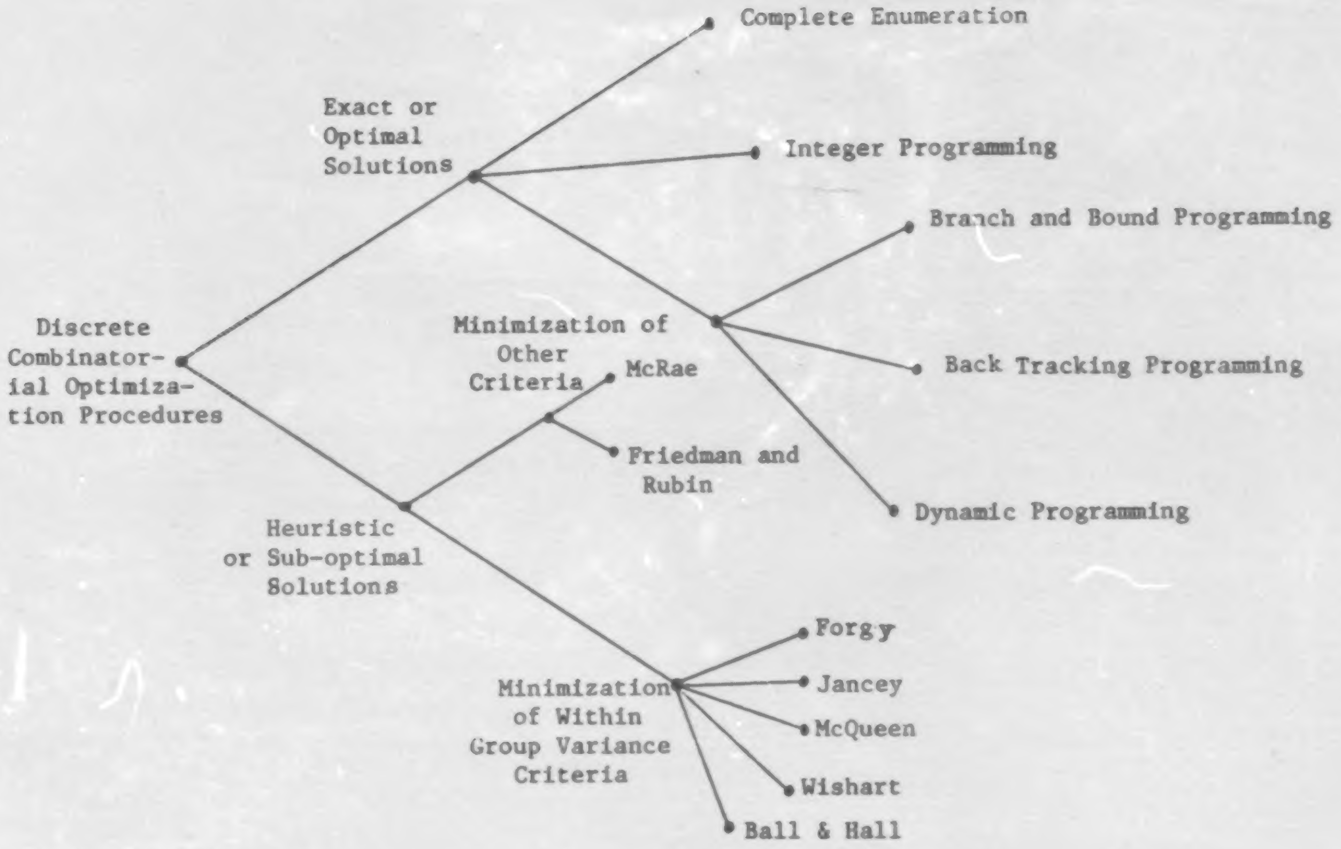

Complete Enumeration
Exact or Optimal Solutions
Integer Programming
Branch and Bound Programming
Discrete Combinatorial Optimization Procedures
Minimization of Other Criteria
McRae
Back Tracking Programming
Friedman and Rubin
Dynamic Programming
Heuristic or Sub-optimal Solutions
Forgy
Jancey
Minimization of Within Group Variance Criteria
McQueen
Wishart
Ball & Hall

Figure 2-1: Combinatorial Optimization Procedures

different groups, is a combinatorial problem whose solution is restricted to integer values (number of students in each group). As such it can, in principle, be solved by complete enumeration of every possible solution. In practice, almost all realistic combinatorial problems are far too large for solution by such simple and direct methods. This is to a very large degree a consequence of the explosive nature of combinatorial problems where the marginal contribution of the nth element to the total effort required to achieve a solution is always exaggeratedly large.

For example, the number of ways that n students can be divided into g distinguishable classes containing $k_1, k_2, \ldots, k_g$ elements, respectively, where $k_1 + k_2 + \ldots + k_g = n$, is

$$\frac{n!}{k_1!k_2!\ldots k_r!} \qquad (2.1)$$

(Eisen, 1969, page 34)

For the relatively small problem of sorting 25 students into 5 groups each of size 5 the number of possible groups is 5.195×10^{12}. Where the size of each group is given as a range, the number of possible groups is even larger. When the size of each group is not restricted (but with each group non-empty), the number of ways of sorting 25 students into 5 groups is a Stirling number of the second kind (Anderberg, 1973, page 3),

$$S_{25}^{(5)} = \frac{1}{5!} \sum_{k=0}^{5} (-1)^{5-k} \binom{5}{k} k^{25} = 2.437 \times 10^{15}, \qquad (2.2)$$

a very large number indeed. It would take an inordinately long period of time to examine so many alternatives.

Numerical combinatorial problems have been traditionally regarded both by mathematicians and applied analysts as highly intractable and complete enumeration of all possible partitions of 150 students into 5 groups (this is an estimation of the magnitude of the problem studied) is infeasible given the present capabilities of computer technology.

Recommendation 1. Complete enumeration of all groupings to identify that grouping which achieves maximal homogeneity should not be considered further, since it is not a feasible procedure.

Until the late 1950's no powerful general methods were available for the solution of discrete-valued optimization problems even though optimization problems involving real-valued variables could be solved in certain circumstances either by classical calculus techniques or by linear programming. It was therefore to be expected that some of the first attempts to derive a general and exact method of solving combinatorial programming problems should have been directed to the problem of integerizing linear programs. More recently certain tree-searching methods have been developed for solving combinatorial problems.

Both integer programming and tree searching methods are often referred to in the operations research literature in the context of the quadratic assignment (QA) problem which involves assigning new facilities to sites when there is an interchange between new facilities. The QA or location problem can be formulated as follows:

Let C_{ikjh} denote the annual cost of having facility i located at site k and facility j located at site h. Also, let the decision variable x_{ik} equal one if facility i is located at site k and equal zero, otherwise. If there are n new facilities and sites, we wish to

$$\begin{array}{ll} \text{minimize} & f(x) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{h=1}^{n} c_{ikjh} x_{ik} x_{jh} \\ \text{subject to} & \sum_{i=1}^{n} x_{ik} = 1, \quad k = 1, \ldots, n \\ & \sum_{k=1}^{n} x_{ik} = 1, \quad i = 1, \ldots, n \\ & x_{ik} = 0 \text{ or } 1, \quad \text{for all } i, k \end{array} \tag{2.3}$$

If facility i is located at site k and facility j is located at site h, then x_{ik} and x_{jh} both equal 1 and the cost term c_{ikjh} is included in the total cost calculation. The first set of constraints ensures that exactly one facility is assigned to each site; the second set of constraints results in each facility being assigned to exactly one site.

The optimal assignment of facilities to locations to minimize costs (analogous to assigning students to groups to maximize homogeneity) is a combinatorial optimization problem and has recently been given exposure in the educational literature by Hubert (1975) who applied the quadratic assignment paradigm as a general data analysis strategy and accordingly interpreted the problem of optimally allocating students to groups as a quadratic assignment problem. The techniques for solving the QA problem are very similar to those utilized in the solution of discrete-valued optimization problems of which the QA problem is one type.

Integer Programming

As a typical pure integer programming problem involving n discrete-valued variables $\{x_1, x_2, \ldots x_n\}$ consider the following:

$$\text{Maximize:} \quad Z = \sum_{j=1}^{n} p_j x_j \qquad (2.4)$$

subject to the m constraints

$$\sum_{j=1}^{n} \alpha_{ij} x_j \leq \gamma_i \quad (i = 1,2,\ldots,m)$$

and the side-conditions $x_j \geqslant 0$

$x_j = \text{integer}$

where p_j, γ_i and α_{ij} are given parameters.

Such problems can be solved by cutting plane methods developed by Gomory (1958). In these methods the problem is first solved by ordinary linear programming, ignoring all integer side-constraints on the variables of the problem. Then new constraints, designated cutting planes, are introduced one by one into the problem. These cutting planes possess the function of progressively eliminating from the total solution space of the given problem, areas which contain real-valued solutions to the problem, but no discrete-valued solutions. By these means, the solution space of the problem is reduced until the optimal integral solution is revealed. Scott (1971, page 8) provided a simple numerical problem and a graphical representation of the cutting planes solution and Vinod (1969, page 506) formulated the grouping problem in integer programming form when the entities are points in a p dimensional Euclidean space.

Practical experience with these cutting plane methods seems to indicate that for small well-behaved problems, these methods will usually converge quite rapidly to the optimal integral solution. Large and complex problems appear to be less amenable to solution, and Scott (1971, page 11) has reported that many cases have been observed where cutting plane methods fail to converge to an optimal feasible solution in a finite number of iterations. Because of the uncertainty of obtaining optimal solutions with integer programming methods and the complex nature of the problem being studied (e.g., eligibility and size constraints on group membership) the following recommendation was made.

Recommendation 2. An integer programming procedure should not be considered further as a viable method of identifying that grouping which achieves maximal homogeneity.

Tree Searching Methods

A more promising approach to the solution of combinatorial programming problems is presented by the family of algorithms known as tree-searching methods. Tree-searching methods are more general than ordinary integer linear programming; they are in addition more specifically related to, and sensitive to, the underlying combinatorial structures of problems containing discrete-valued variables. Specific variations of these algorithms are identified severally as the branch and bound algorithms, backtrack programming, and discrete dynamic programming. These methods all have in common the property that they involve a systematic search over a combinatorial tree for an optimal solution.

A combinatorial tree may be represented as a logical branching process defined over a set of integer variables. Suppose, for example, that a problem contains exactly four zero-one variables, λ_1, λ_2, λ_3, and λ_4. Then every possible solution for this problem may be represented by the tree structure depicted in Figure 2.2, where each vertex in the tree represents a particular and unique solution. The list of variables attached to each of these solution vertices is that sub-set of variables out of the total set of variables whose values are all equal to unity. If inclusion is made of the null solution, which is represented by the origin of the tree, there is a total of $2^4 = 16$ different solutions, whether feasible or not, within the tree.

The principles underlying the orderly construction of the tree are to a large degree self-evident from a consideration of Figure 2.2. However, it should be noted especially that the tree develops by expanding out from the origin into a series of subsequent generations, and that, in general, each vertex on the tree in any generation, t, gives rise to a fan of immediate descendents in

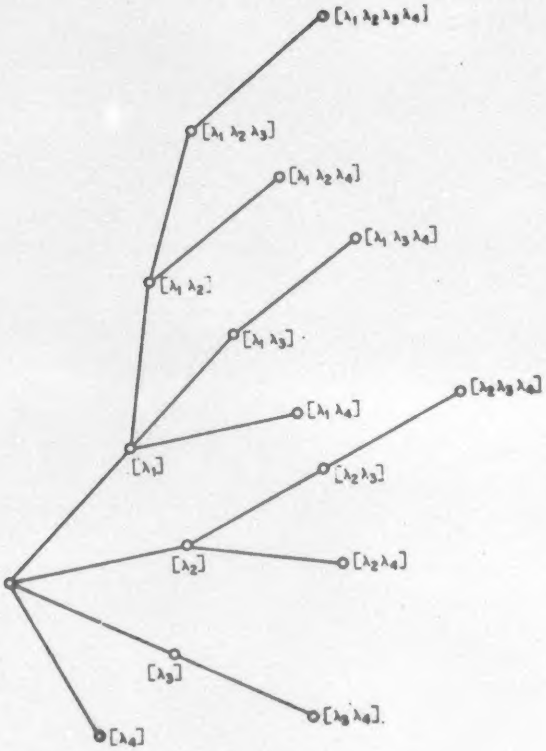

$[\lambda_1 \lambda_2 \lambda_3 \lambda_4]$
$[\lambda_1 \lambda_2 \lambda_3]$
$[\lambda_1 \lambda_2 \lambda_4]$
$[\lambda_1 \lambda_3 \lambda_4]$
$[\lambda_1 \lambda_2]$
$[\lambda_1 \lambda_3]$
$[\lambda_2 \lambda_3 \lambda_4]$
$[\lambda_1 \lambda_4]$
$[\lambda_1]$
$[\lambda_2 \lambda_3]$
$[\lambda_2]$
$[\lambda_2 \lambda_4]$
$[\lambda_3]$
$[\lambda_3 \lambda_4]$
$[\lambda_4]$

Figure 2-2: Combinatorial Tree

generation t + 1. Any one solution in generation t + 1 differs from its immediately antecedent solution in generation t by possessing a single extra entity. This means that it is possible to move systematically forwards and backwards through the tree by adding or dropping, as the case may be, an entity at a time. However, whenever the entity λ_n (e.g., λ_4 in Figure 2.2) occurs as a positive element of any solution, then a terminal state is encountered, and no further branches may be extended out of the corresponding vertex on the combinatorial tree.

The algorithms which are described below represent sets of rules for effecting an intelligently structured search over a combinatorial tree. These sets of rules have in common the property that they are concerned with the progressive transformation of a vector $\{\Lambda\} = \{0, 0, \ldots, 0\}$ denoting the null solution, in the direction of the vector $\{\Lambda^*\} = \{1, 1, \ldots, 1\}$ denoting a fully developed solution. These algorithms progress from an examination of solutions characterized by low and infeasible values of Z, the objective function, to examiniaton of solutions characterized by relatively high and feasible values of Z. At the same time, the searching mechanism representing the transformation process $\{\Lambda\} \leftarrow \{\Lambda^*\}$ progressively partitions the underlying combinatorial tree into sectors where the optimal solution to the given problem may or may not occur. By discarding large segments of the tree where the optimal solution is inferred not to be (a process designated implicit enumeration), these tree-searching algorithms will finally converge upon and identify the globally optimal solution to the problem. There are three principal variations of tree-searching algorithms, namely branch and bound programming, backtrack programming, and discrete dynamic programming.

A Branch and Bound Algorithm

Let Z* denote the optimal objective function for some minimization problem which is to be solved by branch and bound programming. The value of Z* lies between some upper bound U and some lower bound L:

$$L \leqslant Z^* \leqslant U$$

At the initiation of the branch and bound algorithm the value of U may be set equal to any arbitrarily high value, and the value of L may be set equal to any arbitrarily low value. Throughout the period of operation of the algorithm these values are gradually altered until they converge upon, and thus identify, the final optimal solution. The branch and bound algorithm can be described in terms of three basic processes or principles.

The algorithm begins by the establishment of a full set of immediate descendent solutions out of the origin of a combinatorial tree. These solutions are then examined to determine whether or not any are feasible. If any solutions are feasible, then the value of the upper bound, U, is changed to the value of the objective function, Z, for the best such feasible solution. This operation exemplifies the first basic principle for the branch and bound algorithm: The value of U is always changed to the best value of Z out of all current feasible solutions, providing, in addition, that the value of Z is less than the already given value of U.

The second basic principle of the algorithm is now applied. This consists of scrutinizing every current solution, whether feasible or infeasible, by comparing it against the value of U. If any solution possesses a value of Z such that $Z > U$, then neither this solution nor any of its descendent solutions can ever be optimal. For since the objective function of the problem is specified as strictly monotonic-increasing, then it follows that all such descendent solutions must also have values of Z greater than U. However, the value of the optimal feasible solution, Z* is in practice and by definition necessarily less than or equal to U. Therefore, any vertex in the combinatorial tree whose solution possesses a value of Z which is greater than U is permanently deleted from the tree, and no further branches may be drawn from this vertex. This feature explains why the restriction of monotonicity is placed upon the objective function of any problem which is to be evaluated.

The third principle of the branch and bound algorithm relates to the definition and utilization of the lower bound, L, which is at this point brought into the problem. The value of L is simply set equal to the lowest value of Z out of all current solutions; and, usually, this particular solution will be infeasible. The solution corresponding to the current value of L is now used as a parent vertex from which a new set of descendent solutions is derived.

Again, these new solutions are examined to determine if any are feasible; and, again, if any are simultaneously both feasible and possess values of Z less than U, then the value of U is changed to the value of Z for the best such solution. Now the entire set of active vertices within the solution process consists of all new vertices, together with all prior vertices remaining in the problem at large. This entire set of vertices is compared in relation to the value of U, and any vertex where $Z > U$ is permanently deleted from the problem. A fresh set of solutions is again derived out of that vertex which identifies the new value of L.

The algorithm now proceeds on in this manner, branching from the least value of Z within the combinatorial tree, and setting bounds on the outward development of the tree by testing each solution in relation to the value of U. These operations may be formalized schematically as in Figure 2-3. Throughout this process, the value of U is constantly decreasing and the value of L is constantly increasing. In brief, each converges from a different direction of the value of Z*. Finally, when the condition L=U is encountered, then the optimal feasible solution has been discovered and the operation of the algorithm is terminated.

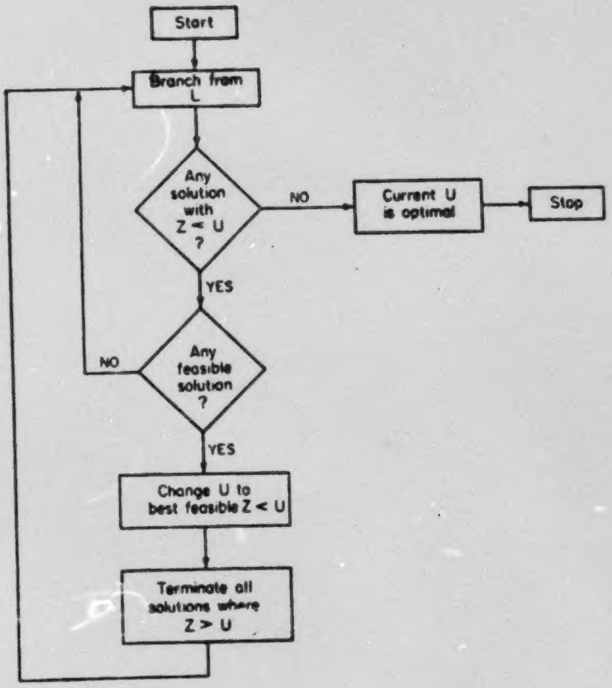

Start
Branch from L
Any solution with Z < U ?
NO
Current U is optimal
Stop
YES
NO
Any feasible solution ?
YES
Change U to best feasible Z < U
Terminate all solutions where Z > U

Figure 2-3: Flow Diagram for the Branch and Bound Algorithm (Scott, 1971, page 17)

The branch and bound algorithm has one highly significant limitation. Because potentially a very large number of solutions must be held in storage as the algorithm progresses, the method (even when modified for computer application in the manner suggested by Scott (1971, page 17)) is suitable only for problems of moderate size which are not likely to explode into an excessively large number of combinatorial possibilities, or where the bounding process is of unusual power. For many problems which would otherwise exceed available storage capabilities, the method of backtrack programming is often more suitable. This method requires only very small amounts of storage. However, this advantage must be paid for by a considerable increase in the number of computations which must be performed to attain a solution.

A Backtrack Programming Algorithm

In contrast to the branch and bound method, the backtrack programming algorithm maintains only one active solution at any one time within the solution process. Thus the total storage requirements of the algorithm are quite small. An upper bound, U, analogous to the upper bound in the branch and bound algorithm, is maintained throughout the operation of the backtrack process. However, no lower bound on the optimal solution is maintained.

The backtrack solution method proceeds by initiating a pattern of search over a combinatorial tree. This pattern of search is directed outwards and in a clockwise direction through the tree. As any vertex is encountered during the search procedure, a decision is made as to whether to continue branching outwards through the tree or whether to backtrack to some previously examined vertex and begin a new branch from that vertex. At any time during the backtrack solution process the major test of whether to continue branching outwards or whether to backtrack is determined by comparing the value of Z for the current solution against the value of U. Whenever the condition $Z < U$ occurs, a test is made to determine whether the associated solution is feasible or infeasible. If the solution is infeasible, the algorithm simply continues branching outwards. If the solution is feasible, however, then a new best upper bound on the value of the optimal objective function has been discovered and the value of U is changed to this value of Z. Thus on the termination of the algorithm, U represents the value of the full optimal solution. Whenever the conditions $Z > U$ occurs, then the solution corresponding to this value of Z together with all its descendents in the combinatorial tree are necessarily non-optimal (since the objective function is monotonic-increasing). Thus whenever a solution yielding the condition $Z > U$ occurs, then the algorithm backtracks into the tree to the nearest node from which a new branch can be drawn. These operations continue until the search process is exhausted. The algorithm as a whole may be decomposed into the schematic representation given in Figure 2-4.

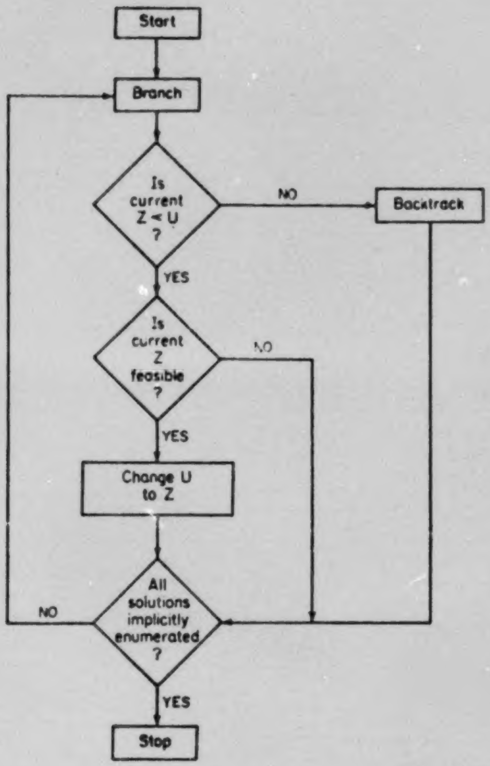

Start
Branch
Is current Z < U ?
NO
Backtrack
YES
Is current Z feasible ?
NO
YES
Change U to Z
All solutions implicitly enumerated ?
NO
YES
Stop

Figure 2-4: Flow Diagram for the Backtrack Programming Algorithm (Scott, 1971, page 23)

Discrete Dynamic Programming

Discrete dynamic programming is a combinatorial programming procedure which partakes of certain characteritics of both the branch and bound and backtrack methods.

The main features of the discrete dynamic programming algorithm may be made concrete by a simple exemplary problem. Suppose that some system is identified as possessing the following characteristics: The system exists within a set of discrete time periods, t = 0, 1,...., T. In any time period, t, the system may be in any one of n states. The number of possible states need not be constant from time period to time period, but only the simplest case of a constant number of states will be considered in this account. At any time, t, the cost of moving from state i to state j is c_{ijt}. At time t = 0 the system is identified explicitly as being in some particular state.

It is now required to compute a time-path for the system such that the aggregate cost of its sequential transitions from state to state between time t = 0 and time t = T is a minimum. Let $Z_t(j)$ denote the cumulated cost of the optimal (least cost) time-path leading up to state j at time period t. This quantity may be determined by the central recursion or branching formula of dynamic programming,

$$Z_t(j) = \min \left[c_{ijt} + Z_{t-1}(i)\right] \qquad (2.5)$$

In other words, $Z_t(j)$ is composed of the two elements, (a) the cost of making the transition from some state i to the particular state j at time t, and (b) the cost of the optimal time-path leading to state i at time t-1. Where the sum of these two elements is minimized over all i = 1, 2,..., n, is given the value of the least cost time-path to state j at time t.

This recursion formula is applied until finally at time t = T the n solutions are determined,

$$Z_T(1), Z_T(2), \ldots, Z_T(n)$$

and the global optimum (Z^*) for the entire problem is

$$Z^* = \min\left[Z_T(i)\right]$$

A simplified flow diagram for the discrete dynamic programming algorithm is shown in Figure 2-5.

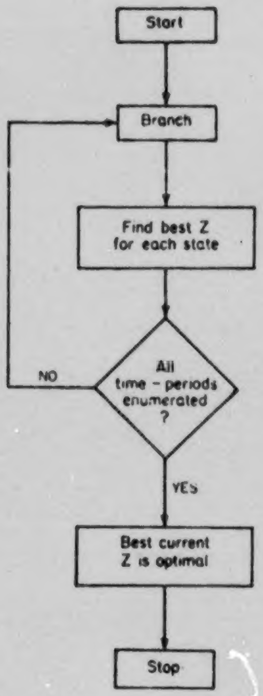

Start
Branch
Find best Z for each state
All time - periods enumerated ?
NO
YES
Best current Z is optimal
Stop

Figure 2-5: Flow Diagram for the Discrete Dynamic Programming Algorithm (Scott, 1971, page 29)

Jensen (1969, page 1034) gave a dynamic programming algorithm for minimizing the within group sums of squares criterion.

General Observations on Tree-Searching Methods

The efficiency of the branch and bound and backtrack algorithms is heavily dependent upon the power of the bounding processes which guide their computational development. To a large degree the efficiency of both algorithms depends upon establishing, even before the initiation of the search process, a strong upper bound of the value of the optimal objective function.

None of these three algorithms is capable of solving large problems, even with the aid of modern electronic computers. Typically, the branch and bound method becomes over-extended in the matter of storage requirements, while the backtrack method becomes over-extended in the matter of solution time. The discrete dynamic programming algorithm has perhaps fewer limitations in these respects than the other two algorithms; however, it is applicable to a much more limited class of problems than either branch and bound or backtrack programming. The combinatorial explosiveness of many problems remains a forbidding obstacle to the application of exact solution methods. It is indeed doubtful if the branch and bound or backtrack programming algorithms could handle any problem with much more than ninety or one hundred variables. Discrete dynamic programming algorithms are most especially sensitive to the number of states in any problem, and computational difficulties become very apparent where this number is in excess of about fifty. A very considerable improvement in the computational efficiency of all of these algorithms is necessary before large problems can be handled with ease.

In the context of the quadratic assignment problem, Francis and White (1974, page 336) noted that implicit enumeration procedures to date have not generally proved to be computationally satisfactory. For example, they estimate that branch and bound procedures are computationally infeasible for the quadratic assignment problem where n is greater than 15. Nugent, Vollmann, and Ruml (1968, page 153), Wagner (1974, page 490), and Hubert (1975, page 54) also noted this restriction on exact procedures and recommended that a generally applicable routine must be based on a method with reasonable cost requirements and great flexibility. The iterative improvement schemes described in the next section have these two characteristics. Although the problem of grouping students for instructional purposes may be formulated in terms of the exact procedures discussed, they appear to be computationally infeasible, which observation led to the following recommendation:

Recommendation 3. None of the exact procedures should be further considered as viable methods for identifying that grouping which achieves maximal homogeneity.

General Solution Procedures for Heuristic Programs

It is often the case that combinatorial problems can be solved only by sub-optimal approximation, where exactness of the final solution is sacrificed for the sake of computational tractability. The solution of programming problems by approximation falls generally within the class of computational methods known as heuristic programming. The number of different heuristic algorithms for the solution of combinatorial programming problems is notably great. Some, such as the algorithm of Echols and Cooper (1968), are developed at a fairly high level of generality. Most heuristic algorithms, however, are designed for the solution of particular problems, and are structured so as to take advantage of the individual characteristics of those problems. Nevertheless, in spite of this apparent heterogeneity, these algorithms tend to have several features in common. In particular, the operating principle underlying almost all heuristic combinatorial programs is one of local optimization. That is, a number of discrete moves is made within the solution process, and each move is optimal within its own frame of reference. The cumulative effect of a set of locally optimized iterations will always guarantee convergence of the solution in the correct direction, and will, further, often yield a final solution which is not far from being fully optimal. Because most heuristic algorithms operate on the principle of local optimization, and because they are also usually constrained by rules limiting the length of any move at any iteration, it is common to categorize these algorithms as steepest ascent Ω-point move algorithms, for Ω = 1, 2, 3,..., although there are some heuristic combinatorial programs which cannot be so simply categorized.

Heuristic programming algorithms tend very generally, (though not without exception) to fall into two classes based on their iterative structure as it relates to the two major types of network systems discussed in the preceding section. In the first case, the algorithm begins with some arbitrary, often infeasible, solution (usually either the null solution $\{\Lambda\}$ or its complement $\{\Lambda^*\}$) and works in the direction of optimality by locally optimized expansion or contraction of the solution set. In the second case, the solution procedure begins with some arbitrary feasible solution and works in the direction of optimality by progressive alteration of the structure or relative positioning of the elements of the solution set. In general, feasibility is maintained throughout this computational process.

Karg and Thompson's (1964) algorithm for the solution of the traveling salesman problem exemplifies the first class and the algorithm devised by Cooper (1963, 1969) for partitioning a point set represents the second. This latter algorithm is more applicable to the problem of grouping students for instructional purposes than is the Karg and Thompson algorithm and is used to clarify the features of the second class of algorithms.

Cooper's Algorithm for Partitioning a Point Set

Suppose there is given a set of n points distributed in the plane. Any point, j, has a Cartesian coordinate set $\{u_j, v_j\}$ which exactly identifies its location. Suppose now that it is desired to partition these n points into m discrete groups in such a way that (a) each group of points has attached to it a centroid (however defined); (b) the location of each centroid is an element of the solution; and (c) the aggregate distance from each point to its associated centroid is a minimum. Mathematically this problem might be expressed as the minimization of

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij} \sqrt{(U_i^* - u_j)^2 + (V_i^* - v_j)^2}$$

subject to

$$\sum_{i=1}^{m} \lambda_{ij} = 1 \qquad (j = 1, 2, \ldots, n)$$

$$\lambda_{ij} = \{^1_0$$

where U_i^*, V_i^* is the coordinate set of the ith centroid. Usually these centroids would be defined either as centres of gravity or as median centres. The zero-one variable λ_{ij} specifies whether or not point j is assigned to centroid i.

The basic principles of the algorithm developed by Cooper are as follows: (a) arbitrarily select a location for each of the centroids; (b) assign each point to its nearest centroid; this operation defines a set of m groups of points; (c) compute a new centroid for each of these groups of points; (d) iterate over steps (b) and (c) until a set of stable groups emerges.

Similar steepest-descent algorithms have been developed to solve the quadratic assignment problem. For example, the steepest-descent Pairwise-Interchange Procedure (Francis and White, 1974, page 338) starts with an assignment, a distance matrix, and a cost matrix, finds from among all pairwise interchanges of facility locations one that causes the greatest decrease in the total cost, revises the assignment, and then repeats the process until the total cost can be no further decreased. Hubert (1975, page 55) noted that many variations of this basic interchange strategy can be defined.

The partitioning clustering methods introduced in Chapter I are very similar to these steepest-descent algorithms. Such algorithms begin with an initial point and then generate a sequence of moves from one point to another, each giving an improved value of the objective function until a local optimum is found.

Concluding Comments on Heuristic Programming

Heuristic programming dispenses with rigor and exactness, and in exchange is characterized by flexibility and practicality. It is to be stressed that it is quite impossible to make any generally valid statement as to the standard of performance of known heuristic algorithms of any type in the solution of very large and very complex problems. All known test problems involving heuristic algorithms have invariably been small in size and well controlled in the matter of internal complications. However, Scott (1971, page 56) noted such tests, for what they are worth, have almost always been highly successful.

At the same time, certain very limited but highly suggestive work by Heller (1960) and Scott (1968) would seem to indicate that mere random sampling will, with high probability, produce solutions to given combinatorial problems within ten to twenty percent of optimality. Almost any well-constructed heuristic algorithm will certainly produce better results than random sampling alone. Thus it is conjectured that many heuristic programs may be inherently compelled to give good results, at least for simple combinatorial problems. However, it is obvious that a great amount of work on the basic structure and configuration of combinatorial programming problems is necessary before any conjecture of this sort can be sustained with confidence. Nevertheless, it may, in addition, be observed that trial and error guided by human judgment and guesswork can sometimes produce good results in the solution of combinatorial problems.

Hubert (1975, page 56) provided a reassuring perspective on the problem of sub-optimal solutions. In optimization applications encountered in operations research, Hubert noted that the failure of a strategy to find global optima is usually of secondary importance if a computationally feasible scheme can still improve upon a solution obtained without the aid of the heuristic. Similarly, in many behavioral science applications the emphasis should be on a procedure's ability to detect or identify an underlying structure in the presence of a reasonable amount of noise. Consequently, the criterion that should be used in evaluating the search strategy is in how well it identifies a reasonably prominent structure and not on how many times it finds the optimal solution for any data set whatsoever. In fact, local optima that are fairly close to a global optimum may be very important substantively in defining alternative representations of a data set; conceivably, local optima may be more important than the single "best" solution (Hubert, 1975, page 56). Considerations such as these led to the following conclusion:

Recommendation 4. The available heuristic algorithms should be reviewed for the purposes of:

(1) selecting one algorithm for implementation, or alternatively,

(2) selecting desirable characteristics of different algorithms to comprise a new algorithm.

Heuristic Programming

Most of these techniques employ three distinct procedures:

(a) procedures for initiating groups,

(b) procedures for relocating entities, and

(c) a grouping criterion.

Consideration of the criterion to be optimized and the selection of an appropriate one will be attempted first, as this may permit the examination of a smaller number of relevant heuristic procedures.

The problem of grouping students for instructional purposes when this grouping is constrained by

(1) the number of groups

(2) the sizes of the groups

(3) student eligibility

is a problem more of dissection rather than classification (Kendall, 1966, page 165). The aim of classification or clustering is to detect in the data, clusters as these naturally occur. In dissection, subdivisions of the data are made on the basis of external constraints and entities are forced into categories because of these constraints and not on the basis of distinctly separated clusters. In the problem being studied groups are to be formed even if there are no distinct clusters presented in the data. It is within rather severe constraints on the formation of groups that group membership is to be maximally homogenized. Thus, the problem of grouping students is not a purely classifactory one.

This distinction is important in deciding upon an appropriate criterion to be optimized. There are two main types of criteria referred to in the clustering literature:

(1) multivariate criteria

(2) the minimum variance criterion.

Multivariate Criteria

Anderberg (1973, page 174) and Everitt (1974, page 27) report three multivariate criteria based on the methods of linear discriminant analysis and multivariate analysis of variance.

(1) Minimization of the ratio of determinants $|W|/|T|$ where T is the total dispersion matrix and W is the matrix of the within groups dispersion. This criterion is widely known as Wilks' lambda statistic. Since the matrix T is the same for all partitions, this criterion is equivalent to minimizing $|W|$.

(2) Minimization of the largest eigenvalue of $W^{-1}B$ where B is the between groups dispersion matrix. This criterion is known as the largest root criterion and is due to S. N. Roy.

(3) Maximization of the trace of $W^{-1}B$. This criterion is known as Hotelling's trace criterion.

Friedman and Rubin (1967), who initially proposed the above criteria recommended them as taking into account the correlations between the variables on which the grouping is based. When variables are highly correlated this set of variables is effectively weighted in comparison to other variables. This feature may be detrimental to the identification of clusters when the other lowly correlated variables are good discriminators. Thus, where the separation out of distinct clusters is important, multivariate criteria may be more appropriate than the total within group sum of squares. Friedman and Rubin (1967, page 1162) also claimed that the multivariate criteria take into account differences in the scaling of the variables. Differences in scaling have a marked affect on measures of similarity and consequently on the composition of groups.

Multivariate criteria, however, are not without their difficulties. First, most of the correlation present is likely to be caused by the existence of the clusters being sought and as Gower (1969, page 360) observed must be observed. What is of paramount importance is the purpose of the grouping and the selection of the variables on which to base the grouping. Given the severe constraints on the grouping, and the purposes of grouping, the extent of the correlation between variables is of reduced importance even though the orthogonality of the variables may be important in the identification of naturally occurring clusters.

Multivariate criteria also do not permit the definition of inter-entity similarity (Friedman and Rubin, 1967, page 1176) which is an important consideration in the placement of individual students. Also the multivariate criteria, by using a pooled within-cluster matrix W, have an underlying assumption that all clusters have the same shape.

Anderberg (1974, page 175) noted that guidelines for making choices among the multivariate criteria are not available (as also observed by Friedman and Rubin (1967, page 1163)). Considering the extra computational cost of solving eigen problems repeatedly at each iteration, it is difficult to identify any clear cut advantages stemming from the use of multivariate criteria in grouping students when this grouping is subjected to very restrictive external constraints and the identification of naturally occurring clusters is not the major goal.

The Minimum-Variance Criterion

The term "minimum-variance" has been used by Forgy to describe the basis of those methods which attempt to minimize the within-group sum of squares. In this context, any method which imposes some form of constraint on the spread, or variance, of clustered points is included in the category. The classical example of this concept is exhibited by Sorensen's method (1948), and the minimum-variance approach is epitomized by his statement as reported in Wishart (1969, (a) page 288) "only one demand may justly be made on the nature of the vegetation in the limited area under investigation, namely that it be homogeneous with as much approximation to that mathematical concept as nature can offer." To impose the requirement that a plant community should exhibit as near total homogeneity as is possible, that is, without any major factor of variation, is probably a perfectly valid constraint in the context of vegetation analyses. As Sorensen went on to say, "the various types of vegetation often are so insensibly merged as to form a sliding scale," and the use of a clustering method which searches for "natural" or "distinct" datum groupings would almost invariably fail to meet the ecologists' demands.

The underlying axiom of variance constraint seems to have been developed intuitively from the idea that a resultant group of individuals should be homogeneous in relation to the total setof variables. That is, each individual should be relatively similar to every other individual in the same cluster for each variable. This method provides for clustering individual entites into groups on the basis of their overall similarity. Expressed in geometric terms, the set of points which constitutes a minimum-variance cluster would be of spherical shape and should not possess any major axis of variation.

The minimum variance criterion is not without its objections and Wishart (1969,(a) page 292) noted that such methods produce clusters which are

(a) modified by changes in the character set,

(b) destroyed by the introduction of non-relevant characters,

(c) sometimes partitioned by artificial and unsatisfactory boundaries.

Objections (a) and (b) are related to the purposes of the grouping and also to the consequent selection of variables on which the grouping is based. Although important considerations for the teacher effecting the grouping, these objections also apply to other criteria. Indeed we would expect, in fact, require any grouping method to be sensitive to such changes in the input. Objection (c) is of reduced significance in the problem of grouping students because of the accompanying administrative constraints which, without any other considerations, would impose critical and unsatisfactory boundaries on the data.

Friedman and Rubin (1968, page 1177) while noting that the minimum trace W criterion (equivalent to the minimum variance criterion) is much less costly in computation time than the other multivariate criteria also noted its major fault as not taking into account the within-group covariance of the measurements (the correlations between the measurements) whereas the multivariate criteria do. However, Hartigan (1975, page 63), Fleiss and Zubin (1969, page 240) and

Cormack (1971, page 326) noted that the correlation structure within clusters may vary considerably from cluster to cluster, so that a pooled covariance matrix is inappropriate. Fleiss and Zubin considered that straightforward correlations between all variables based on the entire sample are worse than useless. Positive within group correlations may end up negative across all groups, negative within groups correlations may end up positive and zero within group correlations may end up far from zero. Rather than correlations across the entire sample, correlations among variables within groups are more appropriate in identifying naturally occurring clusters. However, such within group correlations are most frequently unavailable and consequently most researchers ignore the problem of correlations completely, although this is hardly a solution. Everitt (1974, page 64) proposed a principal components analysis on all variables prior to the cluster analysis which then employs only the first few principal components. Although this line of attack was considered promising, (Fleiss and Zubin, 1968, page 243) these critics also noted examples where principal component analysis performs rather poorly.

Because (1) the purpose of grouping students was to produce groups of individuals maximally homogeneous in relation to the total set of variables and also to ensure that each individual was relatively similar to every other individual in the same cluster for each variable,

(2) the grouping was less oriented towards the objectives of classification or clustering than it was towards the purposes of dissection, and

(3) because of the administrative restrictions in the formation of groups, the following recommendation was made:

Recommendation 5. A minimum variance criterion should be used as part of a heuristic-programming technique.

Minimum Variance Procedures

The following twelve methods are merely a representative selection from a much larger list of minimum variance procedures. After considering these twelve methods, the most relevant were selected for further examination, the results of which examination were used to develop a profile of an acceptable algorithm useful for grouping students. A decision was then made as to whether any existing algorithm had the capabilities required to solve the problem.

1. Sorensen (Complete Linkage, 1948).

A group of individuals comprises a cluster provided that no two individuals have a similarity which is less than a critical user threshold r. Using d^2 this is interpreted to mean that the maximum distance between any two cluster points must not exceed the threshold, that is, the threshold defines the maximum permitted diameter of the cluster subset.

2. MacNaughton-Smith (Furthest Neighbor, 1965).

Each of the N individuals is originally designated as a single-point cluster, and a hierarchy is defined by a sequence of (N-1) fusion steps for which, at each step, those two clusters having the smallest resultant diameter at union are combined. The hierarchy obtains all the possible groupings which can be derived by Sorensen's method for any threshold value.

3. Ward (Error Sum, 1963).

The "error sum of squares objective function" is defined as the within-group sum of squares, or the sum of the squared distances from each point to its parent cluster. The method is defined as a hierarchical process combining those two clusters whose fusion causes the least increase in the objective function at each step.

4. Sokal and Michener (Weighted Average/Centroid, 1958).

The similarity relation between two clusters is measured by the squared distance between their centroids D^2_{PQ}, and the method can be defined as a hierarchical system which combines those two clusters, having minimum D^2_{PQ}, at each of (N-2) fusion cycles.

5. Sokal and Michener (Pair Group, 1958).

The relationship between a single individual i and a cluster P is defined as the average of the similarities between the individual and all the cluster elements. In a sequence of growth cycles, that individual for which this value is a minimum is fused to the cluster concerned.

6. Bonner (Method III, 1964).

A critical distance threshold r is chosen, and an individual selected at random is used as a starting point. The first cluster consists of those points which lie within a sphere of radius r about the starting point. From the remaining points, another is chosen at random to initialize the second cluster, and allocation proceeds as before. When all the points are allocated to clusters, each is reallocated to its nearest cluster center to form disjoint groups.

7. Hyvarinen (1962).

The process is identical to Bonner's except that, rather than choosing random individuals, "typical" points are selected to initialize cluster centers, and the final clusters are defined at allocation time. An information-loss statistic is used to detect "typical" individuals, but in the context of d^2, that point nearest the centroid of the residual set might suffice.

8. Ball and Hall (1965).

K individuals, selected at random, initiate cluster centers, and then each of the remaining individuals is allocated to its nearest center. The cluster centroids are computed and any two clusters P and Q are fused if D^2_{PQ} is less than a user threshold. Also, clusters are split if the variance S^2_x in any one dimension x exceeds another threshold S^2. The cluster centroids replace the original centers, and the method reallocates each datum afresh, and iterates to convergence.

9. MacQueen (1966).

K random individuals are selected to initialize cluster centers. The distance from each datum to its nearest cluster center is computed, and the point is allocated to that cluster if the distance does not exceed a threshold r; when the distance exceeds r, then the point initializes a new cluster centre. At each allocation, the new cluster centroid is computed and replaces the original cluster centre, and when the distance between two centroids becomes less than another limit, the clusters are fused. The process iterates until convergence, and final clusters have the diameter constraint 2r.

10. Sebestyen (1962).

This method resembles MacQueen's allocation algorithm with the exception that two thresholds are selected (defining spheres of radius r and R about the cluster centers, r R). If the distance from a datum to its nearest center is less than r it joins that cluster, if the distance is greater than r but less than R, the datum is set aside and allocated at a later iteration, and if the distance exceeds R then the point initializes a new cluster center. Cluster diameters are therefore constrained to 2R.

11. Jancey (1966).

Rather than select k random individuals, Jancey selects k random points for centers and allocates each datum to its nearest cluster center. When all the points have been allocated, the cluster centroids are computed, and the centers are moved to new positions relative to the centroids. The method then returns to reallocate and iterates to convergence. Jancey proposes an "over-relaxation parameter," to determine the new cluster centers after reallocation, which he claims speeds up the approach to equilibrium. However, the result, at convergence, is that the final cluster centers are sited at their centroids. As such, the method does not have any marked diameter constraint.Jancey goes on to propose that different values of k should be tested, the optimum solution being obtained when the total within-group variance is minimized (the definition of total within-group variance is exactly the same as Ward's error sum of squares).

12. Forgy (1965).

In search of the ideal minimum-variance solution, Forgy adopts Ward's hierarchical process to obtain a part-optimum solution for k clusters and then proceeds to reallocate cluster individuals to their nearest cluster centers. After this, he tries "sliding the partitions back and forth between each pair of centroids" in an attempt to improve the error sum of squares. The final groupins are very similar to those obtained by Jancey's method.

Some of the above minimum variance methods are hierarchical and therefore it is proposed not to discuss them further. Rather, the partitioning procedures of Forgy, Jancey, MacQueen, and Ball and Hall as well as variants on them proposed by Wishart and McRae will be examined. The discussion is presented within the framework of

(a) initial configurations

(b) nearest centroid sorting.

Initial Configurations

The methods discussed here begin with an initial partition of the data units into groups or with a set of seed points around which clusters may be formed.

Seed Points

A set of k seed points can be used as cluster nuclei around which the set of m data units can be grouped. The following methods are representative examples of how such seed points can be generated and were provided by Anderberg (1973, page 157).

1. Choose the first k data units in the data set (MacQueen, 1967). If the initial configuration does not influence the ultimate outcome in any important way, then this method is the cheapest and simplest.

2. Label the data units from 1 to m and choose those labeled m/k, 2m/k,...,(k-1)m/k, and m. This method is almost as simple as method 1 but tries to compensate for a natural tendency to arrange the data units in the order of collection or some other nonrandom sequence.

3. Subjectively choose any k data units from the data set.

4. Label the data units from 1 to m and choose the data units corresponding to k different random numbers in the range 1 to m (McRae, 1971).

5. Generate k synthetic points as vectors of coordinates where each coordinate is a random number from the range of the associated variable. Unless the data set "fills" the space, some of these seed points may be quite distant from any of the data units.

6. Take any desired partition of the data units into k mutually exclusive groups and compute the group centroids as seed points (Forgy, 1965).

7. An intuitively appealing goal is to choose seed points which span the data set, that is, most data units are relatively close to a seed point but the seed points are well separated from each other. Astrahan (1970) strives for this goal by using the following procedure:

a. Compute the "density" for each data unit as the number of other data units within some specified distance, say d_1;

b. Order the data units by "density" and choose the one with the highest "density" as the first seed point;

c. Choose subsequent seed points in order of decreasing "density," subject to the stipulation that each new seed point be at least a minimum distance, say d_2, from all other previously chosen.

Continue choosing seed points until all remaining data units have zero "density," that is, they are at least a distance of d_1 from every other data unit. Assuming that an excess of seed points are generated by this method, hierarchically group the seed points until there are just k such points.

The choice of the d_1 and d_2 parameters may require good judgment or several guesses; if d_1 is chosen too small there may be many isolated data units with zero density whereas if d_1 is too large a few seed points will cover the entire data set. In general, d_2 should be larger than d_1; unless d_2 is at least twice d_1 some data units may contribute to the density value of more than one of the chosen seed points. The elaborate nature of the method makes it more expensive than the alternatives.

8. Ball and Hall (1967, pages 72-74) suggest a somewhat simpler approach than that used in method 7 above. Take the overall mean vector of the data set as the first seed point; select subsequent seed points by examining the data units in their input sequence and accept any data unit which is at least some specified distance, say d, from

all previously chosen seed points; continue choosing points until k seed points are accumulated or the data set is exhausted. This method is sufficiently simple that two or three values of the threshold distance d could be tried if the first value gave too few seed points or examined too little of the data set.

This list of methods certainly is not exhaustive, but it does provide a setting for a few observations. First, methods 1, 2, 3, 4, and 8 all share the property that every seed point is itself a data unit, and therefore any cluster built around such a point will have at least one member; the seed points from method 5 easily could result in one or more empty clusters, whereas methods 6 and 7 are relatively immune to such oddities. Second is the topic of randomness; methods 1, 2, 4, and 5 have elements of randomness about them, either through an implicit assumption of random ordering of data units within the data set or through explicit random selection. Third, indifference may be case aside in preference to a deliberate effort to span the data set with seed points as in methods 7 and 8. Such methods seem less prone to giving distorted or badly balanced configurations than are methods involving random selection. Methods like 7 and 8 are, however, computationally more expensive.

Initial Partitions

In some clustering methods, the emphasis is on initially generating a partition of the data units into k mutually exclusive clusters. Some methods of generating such partitions are considered below. In several of these methods, a set of seed points is used.

1. For a given set of seed points, assign each data unit to the cluster built around the nearest seed point (Forgy, 1965). The seed points remain stationary throughout the assignment of the full data set; consequently, the resulting set of clusters is independent of the sequence in which data units are assigned.

2. Given a set of seed points, let each seed point initially be a cluster of one member; then assign data units one at a time to the cluster with the nearest centroid; after a data unit is assigned to a cluster, update the centroid so that it is the true mean vector for all the data units currently in that cluster (MacQueen, 1967). The cluster centroids migrate so the distance between a given data unit and the centroid of a particular cluster may vary widely during the assignment process; accordingly, the resulting set of initial clusters is dependent on the order in which data units are assigned. MacQueen's suggestion of using the first k data units as seed points permits the assignment process to begin with the data unit numbered k + 1; therefore, it is unnecessary to be concerned with the possibility of using a data unit twice, once as a seed point and once in the assignment process.

3. In most cases, a hierarchical clustering method can produce an excellent initial partition. Wolfe (1970) used the Ward Hierarchical Clustering Method to provide an initial set of clusters for his algorithm. However, a complete hierarchical clustering of the entire data set may require more effort than the rest of the analysis and certainly tends to limit the size of the problems than may be considered.

4. Various random allocation schemes could be used. For example, assign a data unit to one of the k clusters by generating a random number between 1 and k. The difficulty with all such random schemes is that the resulting groups are spread more or less uniformly over the entire data set and their centroids are k different estimates of the data set mean vector. Such groups have no properties of internal homogeneity and are not clusters at all. In general, random allocation to groups is not an attractive alternative.

5. The user could use his judgment to sort data units into an initial partition. Because the specifying of seed points leads to an initial partition and because most of the methods reviewed for establishing initial partitions use seed points anyway, it is recommended that the preferred procedure begin with a set of seed points. This procedure (a) should scan the whole data set and (b) take into account the "density" or compactness of the data units in the data set, thereby being less likely to result in distorted or badly balanced configurations than are methods involving random selection.

Recommendation 6. Seed points leading to an initial partition should scan the whole data set and take into account the density of the data set.

Nearest Centroid Sorting

A set of seed points can be computed as the centroids of a set of clusters, and a set of clusters can be constructed by assigning each data unit to the cluster with the nearest seed point. The simplest iterative clustering methods merely consist of alternating these two processes until they converge to a stable configuration. Presented below are several such methods suited to the basic problem of sorting the data units into a fixed number of clusters such that every data unit belongs to one and only one cluster.

Forgy's Method and Jancey's Modification

Forgy (1965) suggested a very simple algorithm consisting of the following sequence of steps:

1. Begin with any desired initial configuration. Go to step 2 if beginning with a set of seed points; go to step 3 if beginning with a partition of the data units.

2. Allocate each data unit to the cluster with the nearest seed point. The seed points remain fixed for a full cycle through the entire data set.

3. Compute new seed points as the centroids of the clusters of data units.

4. Alternate steps 2 and 3 until the process converges; that is, continue until no data units change their cluster membership at step 2.

Jancey (1966) independently suggested the same method except for a modification at step 3. The first set of cluster seed points is either given or computed as the centroids of clusters in the initial partition; at all succeeding stages each new seed point is found by reflecting the old seed point through the new centroid for the cluster. Figure 2-6 illustrates the process. The line from point 1 to point 2 may be viewed as an approximation to the local gradient, the direction in which the seed point should move for greatest improvement in the partition. However, since data units were assigned to the cluster on the basis of their proximity to point 1 rather than point 2, it

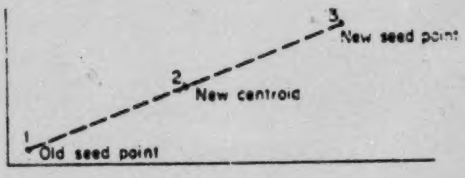


Figure 2-6: Jancey's Seed Point Reflection Method

might be inferred that the movement of the centroid was retarded, and therefore the new seed point should overshoot the computed centroid. Jancey suggested that this technique will accelerate convergence and possibly lead to a better overall solution through bypassing inferior local minima.

Since the seed points are recomputed only after the full data set has been reallocated, the results of these two methods are not affected by the sequence of the data units within the data set.

MacQueen's k-Means Method and a Variant. MacQueen (1967) used the term "k-Means" to denote the process of assigning each data unit to that cluster (of k clusters) with the nearest centroid (mean). The key implication in this process is that the cluster centroid is computed on the basis of the cluster's current membership rather than its membership at the end of the last relocation cycle as with the Forgy and Jancey methods. MacQueen's algorithm for sorting m data units into k clusters is composed of the following steps:

1. Take the first k data units in the data set as clusters of one member each.

2. Assign each of the remaining m - k data units to the cluster with the nearest centroid. After each assignment, recompute the centroid of the gaining cluster.

3. After all data units have been assigned in step 2, take the existing cluster centroids as fixed seed points and make one more pass through the data set assigning each data unit to the nearest seed point.

The last step is the same as the Forgy method except that the reallocation phase is performed just once rather than being continued until convergence is achieved.

By using the first k data units as seed points and relying on only one reallocation pass, this method is the least expensive of all clustering methods discussed.

However, blindly using the first k data units may be less than satisfactory unless the user can arrange to place his choices for the initial centroids at the front of the data set.

The set of clusters constructed in step 2 of the algorithm depends on the sequence in which the data units are processed. MacQueen (1967, page 290) reported some preliminary investigations into this effect. His experience indicates that the ordering of the data units has only a marginal effect when the clusters are well separated; differences from one ordering to the next are due largely to ambiguities arising from data units which fall between clusters. MacQueen also reported that he tried three different orderings when grouping a set of 250 data units into 18 clusters; the within group error sum of squares differed by at most 7% among the three sets of clusters.

The economy of effort inherent in this method stems from acceptance of the first reallocation of data units as opposed to continued processing until convergence is achieved. Apparently the method gives useful results because most major changes in cluster membership occur with the first reallocation; subsequent reallocations usually result in relatively few reassignments.

A convergent clustering method using the k-means process can be implemented through the following sequence of steps:

1. Begin with an initial partition of the data units into clusters. If desired, the partition could be constructed by using steps 1 and 2 of the ordinary MacQueen method, though any of the methods given earlier could also be used.

2. Take each data unit in sequence and compute the distances to all cluster centroids; if the nearest centroid is not that of the data unit's parent cluster, then reassign the data unit and update the centroids of the losing and gaining clusters.

3. Repeat step 2 until convergence is achieved; that is, continue until a full cycle through the data set fails to cause any changes in cluster membership.

This convergent k-means process is a basic element of Wishart's RELOC and McRae's MIKCA computer programs (Wishart, 1970, page 45 and McRae, 1971,(c) page 6).

Convergence

Of the nearest centroid methods, Jancey's method cannot be shown to be convergent (Anderberg, 1973, page 162) and MacQueen's k-means method involves only two passes and hence does not continue to convergence. However, the Wishart and McRae variants of the k-means are convergent, as is the Forgy algorithm.

The criterion chosen for deciding convergence of these clustering methods is stability of cluster membership; an alternative criterion is stability of the cluster seed points. These two criteria are equivalent for the Forgy and convergent k-means methods because the seed points are the cluster centroids which are dependent only on the cluster memberships.

A broad outline of the convergence proof for the Forgy and Wishart, McRae k-means algorithms involves the following steps:

1.

$$W_k = \sum_{j=1}^{k} \sum_{i=1}^{n} (x_{kji} - \bar{x}_{ik})^2 \qquad (2.7)$$

W_k denotes the error sum of squares for cluster k. For a given partition of a data set into h clusters, the total within group error sum of squares is

$$W = \sum_{k=1}^{h} W_k, \qquad (2.8)$$

and W has a characteristic value for the partition. Note that $\sum_{i=1}^{n}(x_{ijk}-\bar{x}_{ik})^2$ is the squared Euclidean distance between the centroid of cluster k and the jth data unit in that cluster.

2. The number of different ways a data set of m data units may be partitioned into h clusters is a Stirling number of the second kind (Chapter II, page 67).

$$S_m^{(h)} = \frac{1}{h!} \sum_{i=0}^{i=h} (-1)^{(h-i)} \binom{h}{i} i^m, \qquad (2.9)$$

which is a finite number, if m is finite. Therefore, any method which generates each partition at most once is finitely convergent because there are only finitely many different partitions. Consider methods in which the current partition is altered only if the change gives a new partition with a smaller total within group sum of squares W. Since each partition has a characteristic value of W, such methods cannot regenerate a partition which was abandoned at an earlier stage and therefore such methods are convergent.

The "convergent k-means" method and Forgy's method can now be shown to be convergent. Let the most recently computed set of cluster centroids be denoted as the seed points. In both methods, a data unit is reallocated only if it is nearer to the seed point of the gaining cluster than to the seed point of the gaining cluster than to the seed point of the losing cluster; if the distance function is chosen as Euclidean distance (or any power thereof or weighted Euclidean distance), then the sum of squared deviations about the seed point decreases more for the losing cluster than it increases for the gaining cluster, thereby giving an overall decrease in sum of squared deviations about the seed points for the partition as a whole. In addition, this sum of squared deviations is decreased even further if it is computed about the new centroids of the clusters rather than the old seed points. Thus, each new partition has a lower value of W than does the partition from which the seed points were computed and therefore these methods are convergent.

The essential difference between the Forgy and k-means method is that in the former the centroids remain fixed for a full cycle through the entire data set, whereas, in the latter, only the losing and gaining clusters have their centroids updated. In both cases, alterations are continued until convergence is achieved.

Both reallocation methods are simply and direct and both seem suitable. However, the k-means method appears to involve more calculations of centroids and distances, and therefore the Forgy reallocation method was preferred. The literature did not contain any information on the comparative efficiency of these two methods.

Recommendation 7: The Forgy reallocation procedure should be used to produce a local minimum of the total within groups sum of squares criterion.

The algorithms observed above utilize nearest centroid sorting with fixed numbers of groups. The literature on cluster analysis, as one would expect, contains similar but more elaborate algorithms which permit the adjustment of the number of groups to conform to this apparent natural structure of this data set. Representative of this set of algorithms are:

(1) MacQueen's k-mean method with coarsening and refining parameters (MacQueen, 1967, page 281)

(2) Wishart's RELOC program which is part of his CLUSTAN 1A package (Wishart, 1971,(c) page 45)

(3) Ball and Hall's ISODATA method (Ball and Hall, 1965, page 1).

Because the problem being examined did not require the added flexibility of variable numbers of groups, these algorithms were not considered further.

Some Implications of Recommendations 5, 6, and 7

The selection of a minimum variance criterion and the Forgy reallocation method necessitates the use of a distance function to express the similarity between pairs of entities. The most commonly used distance measure and the most familiar is the Euclidean metric where the distance between points i and j denoted by d_{ij} is defined as

$$d_{ij} = \left\{ \sum_{k=1}^{h} (x_{ik} - x_{jk})^2 \right\}^{\frac{1}{2}} \tag{2.10}$$

where x_{ik} is the value of the kth variable for the ith entity.

Euclidean distance may be very unsatisfactory since it is badly affected by changing the scale of a variable. Even when all the variables are uniquely determined except for scale changes, Euclidean distance will not even preserve distance rankings. Because of this, variables are frequently standardized before employing Euclidean distance by taking $z_{ik} = (X_{ik})/(\sigma_k)$, or $\frac{X_{ik} - \bar{X}_k}{\sigma_k}$ where σ_k is the standard deviation of the kth variable. Although this has problems, the Euclidean distance calculated from the standardized variables will preserve relative distances.

The principal idea in equalization is to remove the artifact of the measurement unit and anchor each variable to some common numerical property. Recognition of this scaling difficulty led to the following recommendation:

Recommendation 8. All variables on which the grouping is to be based should be standardized.

The importance of each variable to the purpose of the grouping is most properly reflected in the user's choice of these variables. It is probably the choice of variables that has the greatest influence on the ultimate results of a grouping. It is important to bear in mind that the initial choice of variables is itself a categorization of the data which has no mathematical or statistical guidelines, and which reflects the user's judgment of relevance for the purpose of the classification. This, of course, could also be said of the entities chosen for study.

Rigid adherence to standardization of variables is tantamount to saying that an increment in standard deviation is equally important for all variables regardless of the purpose of the grouping.

Apart from their inclusion or omission from the grouping, variables may be accorded more or less importance by applying weights so that agreement with respect to these variables counts for more or less than agreement on others. Although the concept of weighting is a contentious matter in the clustering literature, it appears to be an acceptable option in this situation. It is a useful option where the user wishes to influence the grouping in a particular manner. This led to the following recommendation:

Recommendation 9. The measure of similarity to be used should be the weighted Euclidean metric

$$d_{ij} = \left\{ \sum_{k=1}^{h} a_k (x_{ik} - x_{jk})^2 \right\}^{\frac{1}{2}} \qquad (2.11)$$

where a_k is the weight attached to the kth variable.

The Euclidean metric is usually employed with data measured on an interval scale.

Scales of Measurement

Criterion V (page 38) provides for the use of data measured on different scales when this is considered relevant to the purposes of the grouping. It will therefore be necessary to convert heterogeneous data into interval form before processing it. Mixed variable data sets can be troublesome when clustering data units. Not only is there the problem of measuring association between data units while using different types of variables, but there is also the problem of weighting the contributions of the different variables. Binary variables present no difficulties since they may be treated directly within the framework of interval variables. Likewise, ordinal variables are not too troublesome, since they may be treated as interval variables by using ranks as scores. The difficult problems arise in the simultaneous use of interval and nominal variables. This case is a most difficult scaling problem and can only be solved by inducing an ordering among the classes as well as an equal spacing of the classes. Although Anderberg (1973, page 127) identified some methods for doing this, the grouping procedure being outlined appears to be restricted to binary variables, variables measured as ranked classes, and variables measured on an ordinal scale or an interval scale. The inclusion of nominally measured variables does not appear to be a serious problem considering the purposes of the grouping as well as considering the factors likely to be used in the grouping (see pages 28-36).

Conclusions

The problem of grouping students for instructional purposes had two major components:

(1) a set of constraints on the groups to be formed,

(2) maximizing the homogeneity of these groups as this is measured by the degree of similarity among chosen characteristics.

Chapter I primarily dealt with identifying the nature of the grouping situation as well as the various constraints. The two most restrictive of which were:

(a) placing in the one group only students eligible for the skill or topic to be studied by that group,

(b) the prior determination of the size of each of the groups to be formed.

Chapter II has focused on procedures which lead to the formation of maximally homogeneous groups. These procedures were introduced independently of any administrative constraints. Typically, investigators who use clustering techniques are not concerned with constraining the data but rather seek to identify freely occurring groups or clusters present in the data. Accordingly, the homogenizing procedure outlined had not been considered within the operating framework imposed by the constraints. Without further development, such an algorithm was inadequate for solving the problem of grouping students.

Most heuristic algorithms are designed to solve particular problems and are structured to take advantage of the individual characteristics of these problems. McRae (1971) appeared to be the only investigator who has applied partitioning techniques to the problem of grouping students for instructional purposes. Although McRae utilized similar clustering techniques to these reviewed here, their application did not involve constraints of the kind found in this present problem.

Because of:

(1) the structure searching purposes of clustering algorithms,

(2) the individualistic nature of heuristic algorithms, and

(3) the lack of applications of computerized grouping procedures in school settings,

it was obviously the case that no available algorithm could be directly employed to solve the grouping problem being investigated. It, however, was the case that some features of these other procedures could serve as the basis of a design for an algorithm useful in grouping students for instructional purposes.

The development of an acceptable algorithm concerned the fitting of a homogenizing procedure within a framework of administrative constraints. It was considered that such an algorithm should comprise the following essential elements (it is assumed the number of groups to be formed is known):

(a) a criterion to be optimized

(b) a measure of inter-student similarity

(c) the determination of seed points around which to form groups

(d) the allocation of students to groups on the basis of learning characteristics

(e) the continued relocation of students to groups to optimize the criterion

(f) the allocation of skills/topics to groups

(g) the allocation of size limits to groups

(h) the imposition of group size constraints

(i) the imposition of eligibility constraints.

Operations (c) through (i) are all directly related to the formation of groups. These operations can be performed in different orders and presumably will produce different results as a consequence of these different sequences. For example, it is possible that one sequence would proceed as follows:

1. The selection and allocation of skills/topics to be studied by each group

2. Allocation of size limits to each group

3. The determination of seed points for each group

4. The allocation and reallocation of students to these seed points to form groups for which they are eligible and which maximizes the criterion of homogeneity

5. The imposition of group size constraints.

Another sequence of elements could be as follows:

1. Determination of seed points

2. Allocation and reallocation of students to seed points to form groups which maximize the criterion of homogeneity

3. Allocation of size limits to each group

4. Allocation of skills/topics to be studied by each group

5. Imposition of eligibility constraints

6. Imposition of group size constraints

These are but two of many permutations of the steps in the grouping procedure. Which premutation yielded the most satisfactory results was the subject of an empirical evaluation, the plan of which is presented in the next chapter. This evaluation plan is preceded by a detailed description of the several algorithms developed.

CHAPTER III

FOUR GROUPING ALGORITHMS AND EVALUATION PLANS

The grouping procedures described in this section have as their specific objectives the forming of groups of students to

(i) maximize the homogeneity of the groups, and

(ii) minimize the number of students omitted from the groups.

The administrative constraints which help determine the profile of an appropriate grouping procedure, together with the basic elements of the procedure, have been identified in Chapters I and II. Consequently, four alternative algorithms have been developed on the basis of this research.

Each procedure is described in terms of its (a) user options, (b) input requirements, (c) sequence of steps, and (d) output. Accompanying the description of each procedure is the rationale for each feature not previously mentioned. Also flow diagrams are provided where they appear to be helpful to the reader. In the descriptions which follow "skill" is used synonymously with "topic" and refers to a small segment of an instructional program, usually pertaining to a limited number of instructional objectives.

The first grouping algorithm (Groupal A)

(1) initially selects skills,

(2) matches group sizes with skills,

(3) allocates eligible students to these groups to maximize their homogeneity, and then

(4) applies other constraints.

The second grouping algorithm (Groupal B)

(1) initially allocates students to groups to maximize homogeneity without any constraints,

(2) then on the basis of these groups, selects skills,

(3) and finally applies other constraints.

The third grouping algorithm (Groupal C) is Groupal A modified to include student eligibilities (weighted) with student characteristics in the assignment of these students to groups.

Student eligibilities are coded 1 for eligible and 0 for ineligible. To be eligible for a group, students must have (i) mastered all prerequisites and (ii) not mastered the skill assigned to the group, either on a pre-test or post-test.

Student characteristics are those variables such as achievement scores or learning style scores on which the homogeneity of the groups is based.

The fourth grouping algorithm (Groupal D) is Groupal B modified to include student eligibilities (weighted) with student characteristics in the initial allocation of students to groups.

For each procedure, each group's profile was developed cumulatively as a set of assignments.

Groupals A and C

skill	skill	skill	skill
	size	size	size
		seed point	seed point
			students
Stage 1	Stage 2	Stage 3	Stage 4

After Stage 4, the constraints on the grouping were applied.

Groupals B and D

seed point	seed point	seed point	seed point
	students	students	students
		skill	skill
			size
Stage 1	Stage 2	Stage 3	Stage 4

After Stage 4, the constraints on the grouping were applied.

In the descriptions which follow, it is assumed that each student's record contains the student's ID number and name, scores on each student characteristic and eligibility data on each of the skills of the instructional program being used.

Groupal A

1. Options

The user specifies whether:

a. eligibility for skills is to be taken into account,

b. student characteristics are to be taken into account,

c. the one skill can be studied by more than one group (i.e., single or multiple usage of skills)

Rationale

Option 1a provides the procedure with greater flexibility by making it applicable to instructional programs without a prerequisite structure but where student characteristics are taken into account. Similarly option 1b applies the procedure to instructional programs with prerequisite structures but where student characteristics are not taken into account. Option 1c recognizes that limitations on the availability of learning materials required in the study of program skills may be an important constraint on the grouping. Multiple usage of skills may provide a greater degree of homogeneity than single usage of skills because when implemented may permit more possible relocations of students.

2. Input

The user specifies:

a. the number of groups to be formed,

b. the size of each group to be formed
   as a range (e.g., 20-25),
   as an exact size (e.g., 20-20),
   or without effective size constraints (e.g., 01-99),

c. the number of students to be grouped,

d. the number of skills to be considered,

e. the skills to be considered,

f. the number of student characteristics to be considered,

g. the student characteristics to be considered,

h. the identity of the students to be placed in different groups,

i. the maximum number of iterations permitted in the relocation process.

Rationale

The rationale for steps 2a to 2g is directly based on the problem as defined on page 56. Step 2h prevents the placing in the same group of those students considered to be incompatible. 2i restricts the number of iterations in the relocation process in case convergence is very slow and costly.

3. Range Test

If a_i = lower limit of group i,
b_i = upper limit of group i,
n = number of students to be grouped,
and g = number of groups, then

for the grouping to be feasible

$$\sum_{i=1}^{g} a_i \leq n \leq \sum_{i=1}^{g} b_i \qquad (3\text{-}1)$$

If condition (3-1) is not met, the diagnostic "group sizes not compatible with total number of students" is printed. However, the grouping procedure is continued.

4. Student Data

a. Calculate the number of students eligible for each skill considered.

b. Calculate the mean of each student characteristic and substitute the relevant mean for each missing unit of data.

c. Calculate the variance and standard deviation of each student characteristic.

d. Standardize each student characteristic.

5. Selection of Skills and Matching of Group Sizes

a. For single usage of skills, select skills in order of greatest eligibility

b. Match group sizes requested with skills selected as a one-to-one correspondence between the group sizes and the number of students eligible for the skills chosen, each ranked in descending order of magnitude. For example, one group to be formed is assigned the skill for which most students are eligible and also is assigned the greatest of the requested sizes.

c. For multiple usage of skills, use the same procedure except after each selection and matching, calculate the remaining eligibility for the skill selected as $(e - a_i)$ where e is the number eligible and a_i is the lower limit of the corresponding size. Consider these new eligibilities (remainders) in subsequent selections and matchings.

Rationale

The correspondence in the rank orderings of eligibilities and group sizes may provide for an acceptably small number of students omitted completely from the grouping as well as reduce the number of relocations of students later in the process. These same two advantages may be enhanced by use of the "multiple usage of skills" option which is also likely to result in greater homogeneity of groups in those cases where more than one group is assigned the same skill.

6. Selection of Seed Points

   a. Where each group is assigned a different skill. For each such group, calculate the mean of each student characteristic (in standard score form) over all students eligible for that group. Each vector of means represents a seed point.

   b. Where more than one group is assigned the same skill. (The following procedure is employed for selecting seed points whenever more than one group can be assigned the same skill.)

   For all students eligible for groups assigned the same skill

   (1) All pairwise distances are calculated.

   (2) The two most distant students, say A and B are identified.

   (3) The range (difference between vectors of scores for A and B) is calculated.

   (4) The median student vector, say M is determined.

   (5) Calculate $|A-M|$ and $|B-M|$.

   Select A if $|A-M| < |B-M|$ (3-2)

   Calculate the distances of all students from the selected end point (A or B) and consider these distances in ascending order.

   (6) Consider each group in descending order of its lower limit.

(7) Determine the number of students in each initial partition as follows:

(a) If n is the total number of students to be grouped,

g is the number of groups to be formed,

a_i is the lower limit of group i, i=i...g

(a_1 = group with greatest lower limit) and

$\sum_{i=1}^{g} a_i$ = total of all g lower limits,

then the number of students in group i (G_i) is given by

$$G_i = n \times \left(\frac{a_i}{\sum_{i=1}^{g} a_i} \right), \quad i=1...g \qquad (3\text{-}3)$$

The first G_1 students of the list established in (5) above form the first initial partition, the next G_2 students form the second initial group, and so on.

(8) Seed points are calculated for each of the g groups as the vector of variable means calculated for students selected into each group.

Rationale

In this "proportional division" procedure, the determination of seed points is independent of the order of the entities and considers all of them. It also considers the density of the distribution in partitioning the set of students proportionally to the sizes of the groups rather than in equal proportion. Because students are selected for these ordered groups in the order of least distance from A, this procedure may result in the selection of well separated seed points and a reduced number of relocations later on in the process.

7. Location-Relocation

a. Calculate the distance of all students to seed points.

b. Allocate all students to seed points to form groups on the bases of least distance and eligibility.

c. Establish an omissions group for students ineligible for any skill chosen.

d. Calculate (a) total distance, (b) total sum of squares within, (c) the mean sum of squares within, and (d) centroid for each group.

e. Use these new centroids to recalculate the distances of all students to these centroids and relocate students as in 7b. Repeat steps 7d and 7e until convergence occurs. Convergence is attained when there is no chance in the total distance for two consecutive iterations.

f. Calculate the number of students assigned to each group. Where this number exceeds the upper limit set for the group, relocate students in the following order:

Using the distances of all students to all seed points as caluclated at the end of the last iteration, select students for relocation in the order of greatest distance from their respective seed points thus retaining the greatest possible homogeneity. Assign these students to groups for which they are eligible in the order of least distance to the new centroid. Assign students selected for relocation and who are ineligible for any other group to the omissions group. The diagnostic "Ineligible for group-because of size constraints" is printed.

Rationale

In cases where the group is overloaded, the number of students identified for relocation is the minimum (i.e. the excess over the upper limit). Students less distant from the centroid but perhaps eligible for other groups are not considered. Rather than make an arbitrary decision involving increasing omissions and decreasing homogeneity, the diagnostic referred to in 7f above is printed and the decision concerning reinclusion is left to the user.

8. Incompatible Students

If incompatible students are indicated and are in the same group, that incompatible student furthest from the centroid is removed and placed in the group with the nearest centroid and for which the student is eligible. Otherwise, the student is placed in the omissions group with the accompanying diagnostic "Removed from group-because of incompatibility with student ---."

9. Output

The profile of each recommended instructional group comprises:

a. Group number,

b. Skill to be taught,

c. Number of students in the group,

d. Student name in alphabetical order,

e. Student ID number,

f. Distance of each student from the mean of the group,

g. The mean, variance and standard deviation of each student characteristic (in standard score form).

The profile of the omissions group comprises:

a. Student name and ID number,

b. Reason for ineligibility,

   (1) not eligible for any skill used,

   (2) not eligible for any skill chosen (the accompanying diagnostic lists those skills for which the student is eligible),

   (3) not eligible because of size constraints,

   (4) not eligible because of incompatibility.

c. The number of students omitted.

A flow diagram of the essential steps in Groupal A is provided. The diagram (Figure 3-1) also indicates all printed output.

Groupal B

1. Options

The user specifies whether:

a. eligibility for skills is to be taken into account,

b. student characteristics are to be taken into account,

c. the one skill can be studied by more than one group,

d. seed points are to be (i) specified by the user or (ii) calculated by the program.

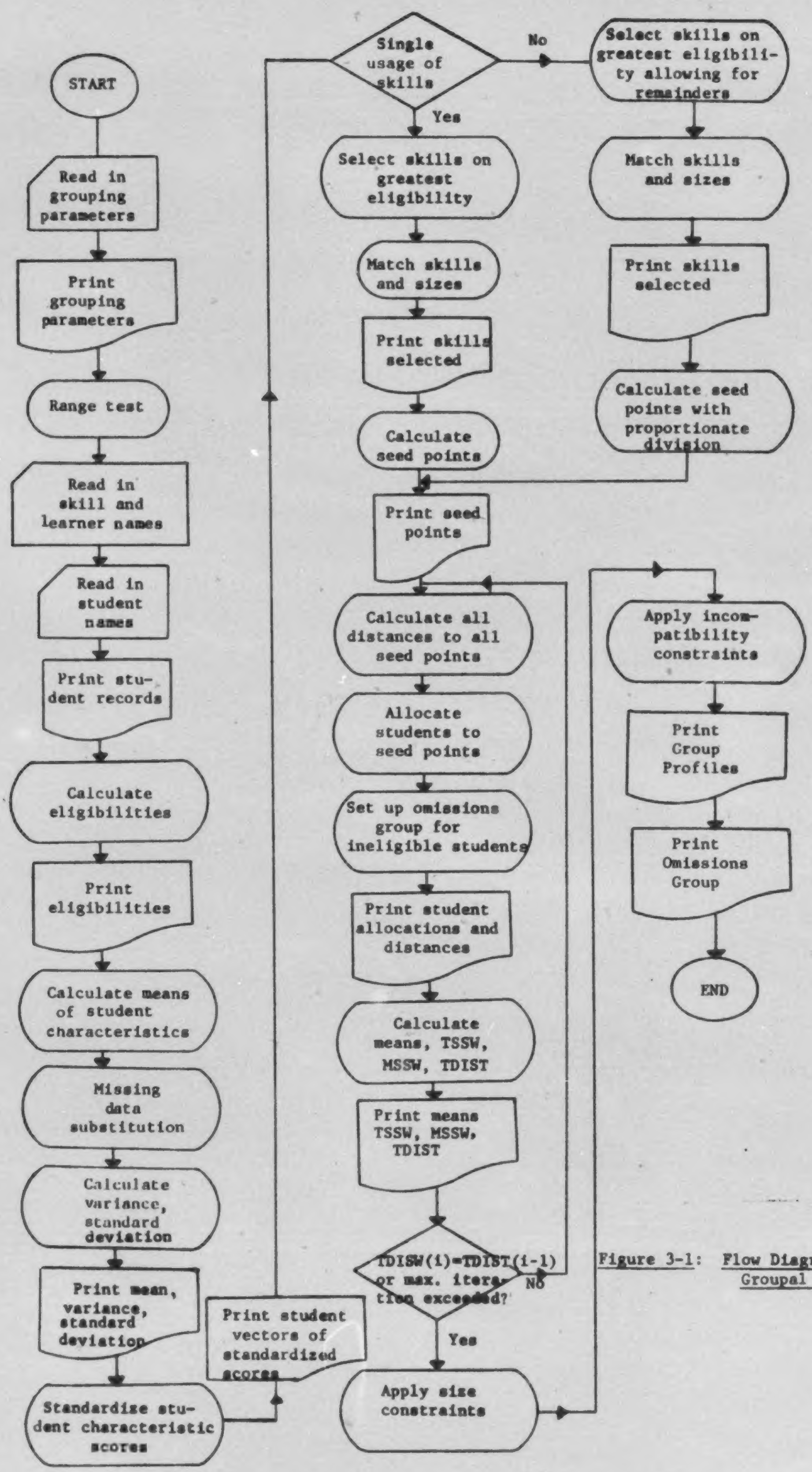

START
Read in grouping parameters
Print grouping parameters
Range test
Read in skill and learner names
Read in student names
Print student records
Calculate eligibilities
Print eligibilities
Calculate means of student characteristics
Missing data substitution
Calculate variance, standard deviation
Print mean, variance, standard deviation
Standardize student characteristic scores
Print student vectors of standardized scores
Single usage of skills
No
Yes
Select skills on greatest eligibility
Match skills and sizes
Print skills selected
Calculate seed points
Select skills on greatest eligibility allowing for remainders
Match skills and sizes
Print skills selected
Calculate seed points with proportionate division
Print seed points
Calculate all distances to all seed points
Allocate students to seed points
Set up omissions group for ineligible students
Print student allocations and distances
Calculate means, TSSW, MSSW, TDIST
Print means TSSW, MSSW, TDIST
TDISW(i)=TDIST(i-1) or max. iteration exceeded?
No
Yes
Apply size constraints
Apply incompatibility constraints
Print Group Profiles
Print Omissions Group
END

Figure 3-1: Flow Diagram of Groupal A

Rationale

Unlike Groupal A in which seed points are calculated using only students eligible for the skill assigned to that group, Groupal B permits the specification of seed points by the user. This feature permits the participation of the user in initially directing the grouping and also allows the evaluation of some alternative methods for determining seed points.

2. Input

As for Groupal A 2a to 2i (page 123) together with

j. the seed points to be used

3. Range Test

4. Student Data

a. Calculate eligibilities.

b. Substitute mean for missing data.

c. Calculate the variance and standard deviation of each student characteristic.

d. Standardize each student characteristic score. (Steps 3 and 4 are the same as in Groupal A).

5. Selection of Seed Points

Determine seed points (if not specified by the user). The same procedure as in Groupal A when more than one group is assigned the same skill (step 6(b) of Groupal A)

6. Location-Relocation

a. Calculate the distances of all students to all seed points

b. Allocate all students to seed points on the basis of least distance.

c. Calculate the total distance, the total sum of squares within, the mean sum of squares within and the centroid for each group.

d. Use these new centroids to recalculate the distances of all students to these centroids and reallocate as in 6b. Convergence is attained when there is no change in the total distance for two consecutive iterations.

e. For each group calculate the number of students eligible for each skill.

7. Selection of Skills and Matching of Group Sizes

Select skills and match group sizes as for Groupal A, step 5.

Apply size constraints as for Groupal A, step 7f.

Apply incompatibility constraints as for Groupal A, step 8.

8. Output

a. The profile of each recommended group.

b. The profile of the omissions group. Step 8 is the same as for Groupal A, step 9.

A flow diagram of the essential steps in Groupal B is provided in Figure 3-2. The diagram also indicates all printed output.

Groupal C

This algorithm is identical to Groupal A other than the following:

1. The list of student characteristics is expanded to include the eligibility data on the skills to be used in the grouping. Eligibility for a skill is coded 1, ineligibility is coded 0. It is this expanded list of student characteristics that is used in the calculation

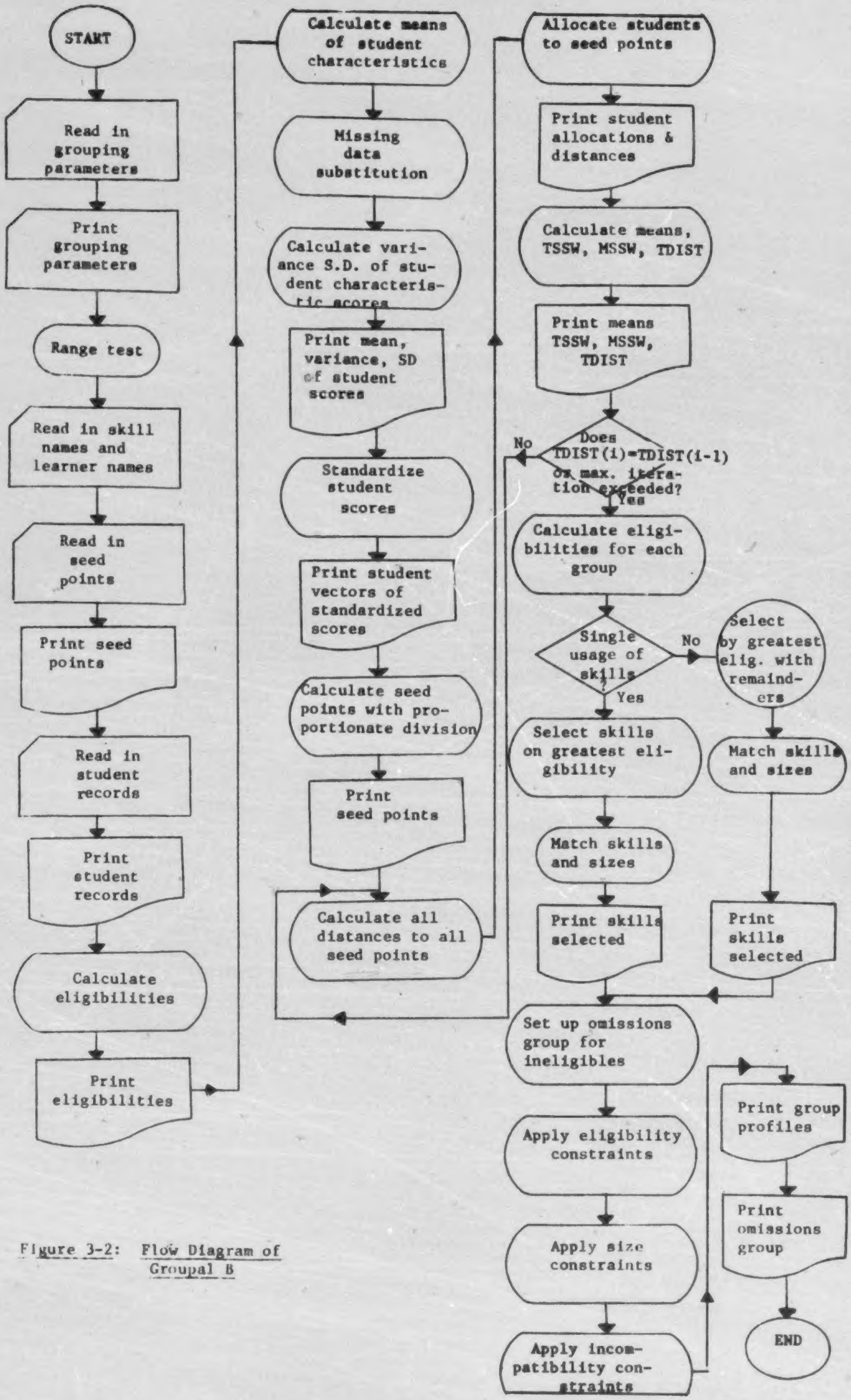

START
Read in grouping parameters
Print grouping parameters
Range test
Read in skill names and learner names
Read in seed points
Print seed points
Read in student records
Print student records
Calculate eligibilities
Print eligibilities
Calculate means of student characteristics
Missing data substitution
Calculate variance S.D. of student characteristic scores
Print mean, variance, SD of student scores
Standardize student scores
Print student vectors of standardized scores
Calculate seed points with proportionate division
Print seed points
Calculate all distances to all seed points
Allocate students to seed points
Print student allocations & distances
Calculate means, TSSW, MSSW, TDIST
Print means TSSW, MSSW, TDIST
No
Does TDIST(i)=TDIST(i-1) or max. iteration exceeded?
Yes
Calculate eligibilities for each group
Single usage of skills?
No
Yes
Select by greatest elig. with remainders
Select skills on greatest eligibility
Match skills and sizes
Match skills and sizes
Print skills selected
Print skills selected
Set up omissions group for ineligibles
Apply eligibility constraints
Apply size constraints
Apply incompatibility constraints
Print group profiles
Print omissions group
END

Figure 3-2: Flow Diagram of Groupal B

of seed points and the allocation-reallocation process of Groupal A. Thus, the major difference between Groupal A and Groupal C is that Groupal C uses the expanded list of learner characteristics which includes the skills eligibility data.

2. The eligibility data on each chosen skill can be weighted and used in the determination of seed points and in the allocation-reallocation process.

For skill i, $$D(x_j, x_k) = \sqrt{\sum_{i=1}^{n} a(x_{ij}-x_{ik})^2}$$

where x_{ij} and x_{ik} are scores on skill i for students j and k,

a is the constant weighting factor applied to all chosen skills,

and there are n chosen skills.

Rationale

The inclusion of skill eligibilities as student characteristics reflects the importance of placing students with similar patterns of eligibilities into the same group. This feature may be enhanced by the selection of the "multiple usage of skills" option which may provide more flexibility in the placement of students. It may be further enhanced by the sole use of skills eligibilities as learner characteristics without the use of any other learner characteristics. Its effective use with other learner characteristics for this purpose is likely to be enhanced by a heavy weighting of each of the skill eligibilities.

In Groupal A, the skills are assigned to groups prior to the assignment of students to these groups and based on maximum eligibilities.

This latter assignment of students to groups may be made more efficient (less later relocations) if used with weightings and with the multiple usage of skills option. Its effect on the homogeneity of the groups in terms of the other learner characteristics is unknown.

Groupal D

This algorithm bears the same relationship to Groupal B as Groupal C does to Groupal A. Its extra features are (a) the inclusion of skills eligibilities with student characteristics and (b) the provision of a weighting option for the original list of skills eligibilities.

Rationale

The inclusion of skills eligibilities as student characteristics and used in forming homogeneous groups before the imposition of eligibility and size constraints can be expected to result in the inclusion in the same groups of students with similar eligibilities. This feature may increase the efficiency of algorithm Groupal B because it is on the basis of these unconstrained groups that the assignment of skills is performed. The formation of these unconstrained groups is then, to some extent, influenced by eligibilities and hence the number of subsequent relocations because of ineligibilities can be expected to be reduced.

The computer programs prepared for each of the four algorithms are presented in Appendices A through D, Pages 306 to 366.

The evaluation plan designed to test the four algorithms is now described.

Evaluation Procedures

The evaluation of the grouping procedures described in this chapter may be considered in two parts. The first part is a comparison of the different grouping procedures developed. This evaluation is designed to answer the first research question: "Which grouping procedure of those compared yields the most homogeneous groupings?" It is this procedure which will be selected for further evaluation.

The second part of the evaluation is a comparison of the selected computerized procedure with a teacher grouping procedure. This evaluation is designed to answer the research questions: "Are the groupings formed on the basis of the numerical grouping procedure more homogeneous than teacher created groups?" "Do teachers involved in the groupings of students perceive the computerized groupings as being a more efficient procedure than those currently employed, as being able to take into account (a) realistic constraints on the formation of groups and (b) relevant student characteristics?"

The complete evaluation plan designed to answer these three questions involved (a) establishing sets of criteria to be used in assessing the performances of the different procedures, (b) designing a testing program in which the different grouping procedures are tested under different conditions and (c) collecting sets of student data on which to test the different procedures. The student data used in the evaluation is now described.

School Setting

The school chosen as the source of student data to be used in Part 1 of the evaluation and as the setting for the teacher assessment of the computerized grouping procedure was a public elementary IGE school which had been a pilot school in the implementation of WIS-SIM for the previous two years. The school's student population was 685 and comprised six instructional units. The number of staff attached to each unit and the number of students in each unit is shown in Table 3-1.

TABLE 3-1

STAFF, STUDENTS BY UNIT

Unit	Grades	Number of Staff Teachers	Intern	Aids	Total	Number of Students
1	K, 1	4	0	1	5	135
2	1, 2	4	0	1	5	90
3	2, 3	4	0	1	5	100
4	3, 4	4	.5	1	5.5	106
5	4, 5	4	.5	0	4.5	96
6	5, 6	4	0	1	5	110
School	K-6	24	1	5	30	637

Units 1 through 5 each studied the Wisconsin Design for Reading Skill Development (WDRSD), Developing Mathematical Processes (DMP), and Science...A Process Approach (SAPA). These instructional programs are objective based programs which were being supported by WIS-SIM. Unit 6, however, did not utilize the DMP program. Current information on each student's achievement status in each program was stored by WIS-SIM.

The Wisconsin Design for Reading Skill Development (WDRSD) includes six elements: Word Attack, Comprehension, Study Skills, Self-Directed Reading, Interpretive Reading, and Creative Reading. Skills in each of the six areas are clustered at levels that correspond generally to traditional grade levels. The six areas and grade level equivalents of the clusters are shown in Table 3-2.

TABLE 3-2

SKILLS BY AREA AND BY TRADITIONAL GRADE LEVEL

Skill Area	K	1	2	3	4	5	6
Word Attack	A	B	C	D	-	-	-
Comprehension	A	B	C	D	E	F	G
Study Skills	A	B	C	D	E	F	G
Self-Directed Reading	A	B	C	D	<----	E	---->
Interpretive Reading	A	B	C	D	<----	E	---->
Creative Reading	A	B	C	D	<----	E	---->

Although the levels are arranged in sequence, the skills within a given level are not necessarily arranged in a hierarchical sequence. Study Skills, for example, has three skills at level A, four skills at level B, eleven skills at level C, fourteen skills at level D, seventeen skills at level E, twelve skills at level F, and ten skills at level G. These 71 skills are also divided into strands or categories of related skills that recur at different levels. The mastery of skills in a strand is frequently a prerequisite for the study of related skills at the next highest level. Formal tests, each keyed to a specific skill, are available for most of the skills in Word Attack, Comprehension, and Study Skills.

DMP is an instructional program for elementary mathematics developed at the Wisconsin Research and Development Center. In DMP, activities designed to promote the attainment of closely related objectives have been clustered to form 90 topics each of which are further subdivided into objectives. Either whole topics or objectives within topics may be prerequisite to the study of later, related topics. The topics are themselves grouped into levels, each level approximating one year of study.

The SAPA program comprises 105 modules, each module being devoted to one of 13 science processes. These modules are separated into 33 stages each containing two to five modules. Modules are also partitioned into clusters which are related by process skill and by content. The modules in any stage can be taught in any order. Within each cluster, instruction proceeds from stage to stage, completing modules at one stage before proceeding to the next. Thus, a module may be a prerequisite for latter modules or itself have prerequisites. When all modules in a stage are completed, students have the prerequisites for the next stage.

Unit 4 appeared representative of the other units in the school in terms of size, staff, and programs. Unit 4 also appeared likely to be able to provide a ready source of student data and was therefore chosen as the basic source of student data required for both Parts 1 and 2 of the evaluation plan. The WDRSD and DMP programs were chosen as the instructional programs to utilize rather than SAPA which had only been very recently introduced.

Part 1 of the Evaluation Plan

The data required for these testing purposes was of two basic types: (a) data relating to the eligibility of students for particular skills of an objective based program and (b) data relating to the characteristics of each student.

Student Eligibility

Eligibility reflects students' status in terms of the prerequisite structure of the instructional program. Ineligibility for a skill is the result of either non-mastery of the prerequisites of the topic or as a result of mastery of that topic (either on a pretest or post-test). It was therefore proposed to utilize three sets of simulated eligibility data for the Study Skills component of WDRSD for fourteen skills C_3 to C_5, C_7 to C_9, D_1 to D_8 with the following characteristics;

(a) 75% average eligibility over all fourteen skills,

(b) 50% average eligibility over all fourteen skills,

(c) 33% average eligibility over all fourteen skills.

Average eligibility was calculated as the total number of students eligible for each skill divided by the total number of skills studied by all students. Eligibility was recorded as either eligible (coded 1) or ineligible (coded 0).

Student Characteristics

Intervally measured data were collected for all students of Unit 4 on:

(1) their learning styles as measured by the Center for Innovative Teaching Experiences (CITE) Learning Styles Instrument (Form C) (Randal, Albright, Babich, Burdine, 1975, page 1);

(2) their achievements as measured by the number of skills mastered in the Study Skills components of the WDRSD program, each skill tested on criterion referenced tests administered throughout 1975-76 (Chester, Askov, and Otto, 1973, page 4);

(3) their scores on the Stanford Diagnostic Test of Reading (Buros, 1975, page 108).

The CITE Learning Styles Instrument (see Chapter 1, page 31) provided measures on each of nine constructs, five in the area of learning, two in the area of working, and two in the area of individual expressiveness. These nine constructs were:

The Area of Learning

1. Auditory Language
2. Visual Language
3. Auditory Numerical
4. Visual Numerical
5. Auditory-Visual-Kinesthetic

The Area of Working

6. Group Learner

7. Individual Learner

The Area of Individual Expressiveness

8. Oral Expressive

9. Written Expressive

The CITE Learning Styles Inventory (LSI) together with the administrative directions are contained in Appendix E. The Learning Styles Inventory was prepared as part of Project CITE in 1974-75 by the Murdoch Teacher Center of the Witchita Public Schools and funded under ESEA Title III. The LSI has not yet published any information on the validity of the instrument but have reported the following measures of reliability for each of the nine constructs obtained from testing 150 elementary students (Randal, Albright, Babich, and Burdine, 1975, page 6).

Visual Numerical had the lowest corrected odd-even coefficient of .2460. Expressive Oral recorded the highest corrected odd-even coefficient of .9520. The item reliability range was from .3682 (Auditory Linguistic) to .9016 (Auditory Numerical). The mode of item coefficients was approximately in the .700 area of frequency and the median was in the .6000 area of frequency. Seventy-three percent of the items were greater than .6000 and eighty-eight percent of the items were greater than .5000.

The formal tests devised for each of the skills in the Study Skills component of WDRSD had demonstrated reliability at a reasonably high level. In general the reliability coefficients reported by Chester, Askov, and Otto (1973, page 4) were .80 or better.

The Stanford Diagnostic Reading Test (Level 2) was designed to diagnose the individual reading difficulties of pupils or to group pupils according to their instructional needs. Scores were in the form of reading ages and the authors reported median split-half reliabilities for Level 1 as .94 and .93 for grades 3 and 4 (Buros, 1975, page 108).

Data Collection

(1) The CITE Learning Styles Inventory (Form C) was administered to one hundred Unit 4 students on March 26, 1976, and to the remaining six students (previously unavailable) on April 2, 1976.

(2) Data on student achievement in Study Skills was also collected from the WIS-SIM files on March 26, 1976.

(3) Stanford Diagnostic Reading Tests results were already available, having been administered in October, 1975.

In all, four sets of data on student characteristics were compiled. Set A, consisting of the learning styles (9 scores), achievement (1 score), and Stanford Diagnostic Reading Test scores (1 score) obtained for all Unit 4 students as reported above. Set B, consisting of randomly generated data for each of the eleven measures, the distribution of each being random normal with each variable having the same mean and standard deviation as for Set A. The Random Number Routine RANNP was used to generate the required distribution.

Set C. A second set of random normal data with each variable having the same mean and standard deviation as for Set A. RANNP was again used. Set D, consisting of Set A with the data for each of the first two students (alphabetically) replaced by a set of extreme scores, one comprising all very low scores, the other comprising all very high scores.

The various data sets utilized in the comparison of the four algorithms are summarized in Tables 3-3, 3-4, and 3-5.

TABLE 3-3

DATA SETS UTILIZED IN THE COMPARISON OF THE ALGORITHMS

Data Set	Eligibility	Student Characteristics
1	75% average eligibility	A
2	50% average eligibility	A
3	50% average eligibility	D
4	75% average eligibility	D
5	33% average eligibility	A
6	75% average eligibility	B
7	75% average eligibility	C

The Testing Program

The testing program designed considered the following elements:

(1) each of the four algorithms,

(2) each of the seven data sets,

(3) single or multiple usage of skills,

(4) different group size constraints,

(5) different procedures for selecting seed points,

(6) different numbers of groups.

TABLE 3-4

NUMBERS AND PERCENTAGES OF STUDENTS ELIGIBLE FOR DIFFERENT SKILLS

Skills Data Sets	1 (C3)	2 (C4)	3 (C5)	4 (C9)	5 (C8)	6 (C9)
Data Set 1 (95%)	77 (73%)	76 (72%)	86 (81%)	75 (71%)	81 (76%)	83 (78%)
Data Set 2 (48%)	52 (49%)	58 (55%)	46 (43%)	47 (44%)	60 (57%)	43 (41%)
Data Set 3 (48%)	52 (49%)	58 (55%)	46 (43%)	47 (44%)	60 (57%)	43 (41%)
Data Set 4 (75%)	77 (73%)	76 (72%)	86 (81%)	75 (71%)	81 (76%)	83 (78%)
Data Set 5 (33%)	40 (38%)	42 (90%)	25 (24%)	32 (30%)	38 (36%)	33 (31%)
Data Set 6 (75%)	77 (73%)	76 (72%)	56 (81%)	75 (71%)	81 (76%)	83 (78%)
Data Set 7 (75%)	77 (73%)	76 (72%)	86 (81%)	75 (71%)	81 (76%)	83 (78%)

Total number of students = 106

TABLE 3-5

DESCRIPTIVE STATISTICS OF STUDENT VARIABLES USED IN THE TESTING PROGRAM

Student Characteristics	Visual Language (VL)	Auditory Language (AL)	Experience Oral (EO)	Experience Written (EW)
Mean	27.26	29.77	27.08	28.43
Variance	45.91	26.21	32.47	35.92
Standard Deviation	6.78	5.12	5.70	5.99
Maximum possible Score	40	40	40	40

If the selection of seed points is limited to three categories (e.g., random selection, teacher selection, and proportional range), the sizes of groups to three (exact, open, typical) and numbers of groups to three (e.g., small, typical, and large numbers of groups) the number of possible combinations of the seven elements is 1,512. Because of the impossibility of testing all combinations and because many such combinations were of little or no importance to the testing of the algorithms, a selection of tests was made. This testing program aimed to provide a comprehensive and realistic testing of the four algorithms as they might be applied in a school setting. The following selections of the available data on eligibility and student characteristics were made:

1. Six of the fourteen skills of the Study Skills component of WDRSD were chosen for possible assignment to groups. These were skills C3, C4, C5, C7, C8, and C9.

2. The student characteristics considered were

   (a) Auditory Language,

   (b) Visual Language,

   (c) Oral Expressive,

   (d) Written Expressive.

3. Both the skills in 1 above and the student characteristics in 2 above were held constant throughout the testing program for Part 1 of the evaluation. This feature permitted comparisons of the effects caused by varying other elements.

Each of the following elements were varied in the testing program:

(a) the data sets,

(b) single or multiple usage of skills,

(c) the number of groups formed,

(d) the sizes of the groups formed.

The testing program decided upon is outlined in Tables 3-6, 3-7 and 3-8.

Table 3-6 refers only to algorithms C and D, which permit the weightings of skills. This table represents 28 tests designed to determine the effects of different weightings on the skills variables. The primary purpose of this set of tests is to select those weightings which produce the best results. It is these weightings which were used in later testings of Groupals C and D.

Both Groupals C and D were subjected to each of the fourteen different tests of Table 3-6. Data set 1 (75% average eligibility and Unit 4 student characteristics) and Data Set 2 (50% average eligibility and the same student characteristics) were selected to provide some variation in the degree of students' eligibilities for the skills considered. The multiple usage of skills option was chosen in all tests because it was considered the option most likely to be used in a school situation and because of its expected effect of permitting more flexibility in the relocation process. Both the number of groups (5) and the group sizes (three groups each of 25-30, one of

TABLE 3-6

TESTS TO DETERMINE THE EFFECTS OF VARIOUS WEIGHTINGS ON SKILLS—APPLIES ONLY TO GROUPALS C AND D

Data Sets	Single/Multiple Usage of Skills	Number of Groups	Group Sizes	Weightings of Skills
1	M	5	25-30, 25-30, 25-30 10-15, 5-10	0.5
1	M	5	"	1.0
1	M	5	"	2.0
1	M	5	"	3.0
1	M	5	"	5.0
1	M	5	"	10.0
1	M	5	"	20.0
2	M	5	"	0.5
2	M	5	"	1.0
2	M	5	"	2.0
2	M	5	"	3.0
2	M	5	"	5.0
2	M	5	"	10.0
2	M	5	"	20.0

10-15, one of 5-10) were selected as being representative of a grouping situation for Unit 4, which includes 5 teaching staff and 106 students. The number of groups and their sizes were held constant for each of the 28 tests.

Seven different weightings were chosen to be applied to the student eligibilities which are considered as student characteristics in Groupals C and D. The weights (0.5, 1, 2, 3, 5, 10, 20) were chosen to provide a sufficiently large range of weights to enable the observation of a trend in the effects of the weightings, if such a trend was to emerge. No other criteria for the selection of weights were used, there being no information available on the likely effects of applying different weights to the skills.

Table 3-7 contains the set of tests used to select one of the grouping algorithms for later comparisons with a manual grouping procedure. The weights chosen for Groupals C and D were those selected on the basis of tests outlined in Table 3-6. Sixteen different tests were applied to each of the four algorithms making 64 tests on which to base a selection of one algorithm for further testing. The rationale for the design of this major section of the evaluation was to provide a comprehensive set of conditions compatible with Unit 4.

TABLE 3-7

TESTING PROGRAM LEADING TO THE SELECTION OF THE MOST EFFECTIVE ALGORITHM

Data Sets	Single/Multiple Usage of Skills	Number of Groups	Sizes of Groups
1, 2, 5, 6, 7	S	5	25-30, 25-30, 25-30, 10-15, 5-10
1, 2, 3, 4, 5, 6, 7	M	5	25-30, 25-30, 25-30, 10-15, 5-10
5	M	8	22-27, 20-25, 17-22, 15-20, 10-15, 8-13, 5-10, 1-5
5	M	3	40-50, 30-40, 20-30
1	M	5	Each 1-99
1	M	5	30, 25, 20, 16, 15

All seven data sets were used in this testing program to give a realistic range of eligibility data, complemented by both real and simulated data on student characteristics, all representative of Unit 4 students.

As was the case in the tests of Table 3-6, the majority of tests involved the multiple usage of skills option. Only 20 out of the 64 tests involved the single use of skills by groups as this was considered to be less representative of a unit grouping situation and also because with a large range of eligibility data available it was expected that some multiple usage options would result in single usage of skills because of the low average eligibilities in some of the data sets (especially data sets 2, 3 and 5).

Three different numbers of groups were used: 3, 5 and 8. Three was considered to be a small number of groups to be formed from a unit of 106 students; eight was considered a large number of groups for that size unit, whereas five was considered to be a typical number of groups to be formed from a unit of 106 students with five instructional staff. Four tests were run using data set 5 (33% average eligibility) and the multiple usage option with each of 3 groups and 8 groups.

Corresponding to these different numbers of groups were different sizes of groups, the total number of students to be grouped being constant. For the 8 group partitions, the size of the groups ranged from a small size of 1-5 to the largest size of 22-27. For the 3 group partition, the sizes of the groups were 20-30, 30-40, and 40-50, much larger groups than for the 8 group partition. One set of four tests was run using a size range of 1-99 for each of 5 groups. This large size range effectively removed the group size constraints. Only four such tests were run (one per algorithm) because this condition was considered unrepresentative of a typical school grouping situation.

Another four tests were run on data set 1 (75% average eligibility) using the multiple usage option and five groups with typical sizes expressed exactly rather than as ranges. It was considered that teachers typically attempt to form groups allowing for some flexibility in size and hence the option of exactly specifying group sizes was not emphasized in the testing program.

The design of the testing program outlined in Table 3-7 involved the selection of a relatively small set of different conditions from a much larger set of possible combinations of conditions. It was considered that the 64 tests selected were a representative sample of conditions in which the grouping algorithms would realistically be implemented.

Table 3-8 refers to a small set of 12 tests designed to assess the influence of different methods of selecting seed points. Groupals B and D provide the options of (a) user specification of the seed points or (b) program determination of the seed points. This set of tests applies only to Groupals B and D and is not part of the testing program on which the selection of one algorithm for later evaluations is based.

Three additional methods of determining seed points were tested for each of Groupals B and D:

(1) random selection of seed points,

(2) selection of the first g data units as seed points, where g is the number of groups formed,

(3) user selection of seed points.

The different sets of seed points and the results obtained from using them were compared with those determined automatically as part of Groupals B and D. The elements of each test are shown in Table 3-8.

TABLE 3-8

DIFFERENT METHODS OF SELECTING SEED POINTS - GROUPALS B AND D

Data Sets	Single/Multiple Use of Skills	Number of Groups	Sizes of Groups	Selection of Seed Points
1	M	5	25-30, 25-30, 25-30, 1 -15, 5-10	1st random selection
1	M	5	"	2nd random selection
1	M	5	"	3rd random selection
1	M	5	"	Teacher selection
1	M	5	"	Selection of 1st 5 students
1	M	5	"	Program selection

Information Collected on Each Test

Descriptive data collected on each of the 104 tests of Tables 3-6, 3-7, 3-8 were:

1. the skills assigned to each group,
2. the students assigned to each group at each iteration,
3. the seed points for each group before the first iteration and after each iteration of the relocation process,
4. the final total distance (weighted Euclidean distance),

5. the total sum of squares within and the mean sum of squares within - after each iteration,
6. the number of iterations,
7. the final group sizes,
8. the number of omissions because of
    a. ineligibility for any skills initially considered,
    b. ineligibility for any skills chosen by the algorithm,
    c. size constraints,
    d. incompatibility constraints.
9. the costs incurred
    a. total run costs
    b. paper costs

Criteria for the Selection of the Most Effective Weights and the Most Effective Algorithm

Two criteria were used as the basis of selection among the weights and among the algorithms:

(1) the average final total distance as a measure of homogeneity (i.e., the final total distance divided by the number of students placed in groups).

(2) the number of students omitted from groups because of ineligibility for the skills chosen by the algorithm or because of size constraints.

Both criteria were considered of equal importance and were therefore accorded an equal weighting throughout both the separate selections of weights and algorithms. All tests of Table 3-6 were accorded equal weighting in the determination of weights and all tests of Table 3-7 were accorded equal weighting in the selection of a single algorithm.

Each algorithm was scored on its performances on each criterion for each test as follows:

(1) Least average distance = a score of 4
    Second least average distance = a score of 3
    Third least average distance = a score of 2
    Greatest average distance = a score of 1
(2) Least omissions = 4
    Second least omissions = 3
    Third least omissions = 2
    Most omissions = 1

The algorithm with the greatest score over all tests and over both criteria was selected for further evaluation.

A similar scoring procedure was used in the determination of the weights to be used with Groupals C and D. In this case, the weights of the ranks were 7 through 1, there being seven weights assessed. In the event of a tie for the first ranking in either selection, the total least CPU time over all tests was to be the deciding factor.

Part 2 of the Evaluation Plan

The set of teacher generated groupings with which the computer generated groupings were compared were performed in the week of May 17, 1976. The leader of Unit 4 chose three curricular areas in which to perform groupings:

(1) Study Skills, a component of WDRSD

(2) Comprehension, a component of WDRSD

(3) DMP

All the parameters of the groupings were chosen by the unit leader to meet her unit's, then current and realistic requirements. The computerized grouping procedure also used these same parameters and the groupings were performed on May 24, 1976. The same descriptive data were collected for the computerized grouping procedure as had been collected in Part 1 of the evaluation.

Data collected on teacher generated groupings were limited to:

(1) Skills assigned to each group,

(2) Final composition of groups,

(3) Seed points (centroids) for each final group,

(4) The final total distance (weighted Euclidean distance),

(5) The final group sizes,

(6) The number of omissions.

The similarity between the three pairs of groupings was assessed by using the mean contingency and chi-square statistics (Hays, 1973, pages 728, 743). The homogeneity of each of the groupings were compared on the basis of average final distance.

Teacher Assessment of the Computerized Grouping Procedure

Teacher assessment of the computerized grouping procedure was obtained by having the five teachers involved in the instruction complete a questionnaire designed for this purpose.

The questionnaire was designed to ascertain teachers' perceptions of (a) the efficiency of the computerized numerical grouping procedure, and (b) its potential to take account of realistic constraints and relevant learner characteristics. The questionnaire, a set of rating scales with provisions made for written subjective comments, was in two parts. Part 1 was designed to identify those features of a computerized grouping procedure which teachers considered important. Part 2 was designed to identify the extent to which teachers perceived the computerized grouping procedure as meeting those features which they considered important. The questionnaire is shown in Appendix G.

Limitations of the Study

(1) The design of the numerical grouping procedures was limited by the review of the techniques undertaken.

(2) The selection of the particular computerized procedure chosen for comparison with the teacher procedure was dependent on the evaluation plan designed.

(3) The application of the computerized grouping procedure was limited by the constraints of the particular problem studied.

(4) The composition of the groupings was dependent on the numerical procedures used.

(5) The composition of the groupings was dependent upon the characteristics chosen as the data to be utilized in the grouping procedure as well as on their precision of measurement.

(6) The meaning of similarity was dependent on the choice of variables, distance function and the algorithms chosen.

(7) The utility of the procedure can only be applied to the particular setting chosen.

This chapter contained descriptions of four algorithms designed in accordance with the recommendations made in Chapter II. The plans for the evaluation of these grouping algorithms were also outlined. The results of these evaluations are presented in Chapter IV.

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TECHNICAL REPORT NO. 399
(Part 2 of 2 Parts)

numerical procedures in the optimal grouping of students for instructional purposes

SEPTEMBER 1976

WISCONSIN RESEARCH
AND DEVELOPMENT
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COGNITIVE LEARNING

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Part 2 of 2 Parts

NUMERICAL PROCEDURES IN THE OPTIMAL GROUPING OF STUDENTS FOR INSTRUCTIONAL PURPOSES

by

Brian F. Lawrence

Report from the Project on
Computer Systems for IGE

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This dissertation is dedicated to Seven Little Australians and their mother, the author's wife.

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ABSTRACT

This study was concerned with the formation of groups of students and specifically addressed the problem: Can a computerized procedure be developed which assigns students to instructional groups, which maximizes the homogeneity of these groups when this homogeneity is based on relevant student learning characteristics, and which takes account of realistic administrative constraints such as eligibility for group membership, sizes of groups, and numbers of groups?

The procedure developed to solve this problem was mathematical in nature and involved utilizing computer technology in its implementation. It aimed to facilitate, in part, the management of a particular individualized program of instruction, namely Individually Guided Education (IGE).

Based on an initial survey of clustering techniques including hierarchical techniques, optimization-partitioning techniques, density-seeking techniques and clumping techniques, a decision was made that the optimization-partitioning techniques applied most directly to the problem being studied. This set of techniques was further surveyed in terms of complete enumeration, implicit enumeration procedures and heuristic procedures which yield local optimal solutions. Despite their disadvantage of yielding sub-optimal solutions, the heuristic partitioning procedures were considered to most closely meet the requirements of the problem.

Four algorithms were designed, each one involving the fitting of a homogenizing procedure within the framework of the administrative constraints of the problem. The homogenizing procedure employed was the Forgy minimum variance partitioning procedure modified by using a proportional division method for selecting seed points and the weighted Euclidean metric as a measure of similarity. The four computer based procedures were evaluated on the basis of their performances on a set of tests which involved varying the parameters of the grouping situation, such as the data on learner characteristics, data on group eligibilities, the number of groups formed, the sizes of the groups, and the single or multiple assignment of instructional topics to groups.

Two equally important criteria were used in the choice of the most effective of the four algorithms--the homogeneity of groups measured on selected learner variables and the number of students omitted from the groups. The algorithm which proved to be most effective was the one which initially assigned instructional topics to groups, matched group sizes with skills, allocated eligible students to these groups to maximize their homogeneity and then applied other administrative constraints.

The effectiveness of this computer based grouping algorithm was further assessed by comparing its recommended groupings with teacher generated groupings when both groupings were subjected to the same constraints. In the comparison performed, the computerized procedure produced much more homogeneous groups than did the teachers and an equivalent number of students were omitted. The profiles of the groups formed by the two methods were noticeably different as determined by the differences in the means of the learner characteristics for each group, a ratio of agreement and the phi coefficient of association.

User perceptions of the efficiency and effectiveness of the computerized grouping procedure were also obtained. The computerized grouping procedure was perceived to be much more efficient in terms of time spent by users in the grouping process than a manual procedure and more efficient than a semi-automated procedure used by the teachers. Respondents, however, mainly gave median ratings of the computerized procedure's success in maximizing the homogeneity of the groups and minimizing omissions from the groups.

The evaluation of the computerized grouping procedure performed as part of this study can only be considered as preparatory to a more comprehensive examination of the effectiveness and efficiency of the computerized grouping procedure. Despite this limitation, it is claimed that the procedure developed warrants this further evaluation.

APPROVED D W Spuck

DATE August 13, 1976

CHAPTER IV

DATA ANALYSES

Presented in this chapter are the findings of the evaluations designed to answer the three research questions. These findings together with their associated analyses are arranged in the following order:

(i) the selection of weights used in the evaluation of Groupals C and D;

(ii) the selection of one of Groupals A, B, C and D for comparison with a teacher grouping procedure;
This relates to Research Question 1: "Which grouping procedure of those compared yields the most homogeneous groupings?"

(iii) The effects of the various options and administrative constraints on the groupings produced by each of the algorithms;

(iv) the comparison of teacher generated and computer generated groupings, which relates to Research Question 2:
"Are the groupings formed on the basis of the computerized grouping procedure more homogeneous than teacher created groups?"

(v) teacher perceptions of the computerized grouping procedure, which relates to Research Question 3:

"Do teachers involved in the groupings of students perceive the computerized grouping procedure as being a more efficient procedure than those procedures they currently employ? The four grouping procedures, described in Chapter III were:

Groupal A, in which students were assigned to groups initially determined by the administrative constraints.

Groupal B, in which groups were initially formed on the basis of student similarities. These groups were then modified to fit the administrative constraints.

Groupal C, which is Groupal A modified to consider weighted skill eligibilities as characceristics in the assignment of these students to student groups.

Groupal D, which is Groupal B modified to consider weighted skill eligibilities as student characteristics in the initial assignment of students to group.

It was with the effects of different weightings of skills eligibilities in Groupals C and D that the first evaluation was concerned.

Selection of the Most Effective Weights

The purpose of this set of tests was to select that weighting of skill eligibilities (considered as student characteristics) which most consistently produced the most homogeneous groups and the least number of omitted students. The rationale for weighting skill eligibilities and for considering them as student characteristics, was that the simultaneous use of student eligibilities and student characteristics might result in a more efficient initial grouping thereby producing less relocations later in the process.

Table 3-6, page 150 details the 14 tests designed to determine, from among seven weights, that one which most consistently yields the most homogeneous groupings and the least number of students omitted.

Tables 4-1, 4-2 and 4-3 show the results of the various weightings for Groupal C; Tables 4-4, 4-5 and 4-6 show the results of similar tests for Groupal D.

For each of the 14 weighting tests the following elements were held constant:

(i) the number of students to be grouped was 106,

(ii) the number of groups formed was 5,

(iii) three groups were each of size (25-30), one was of size (10-15) and the other was of size (5-10),

(iv) the multiple usage of skills option was chosen.

Table 4-1 shows the results obtained using data set 1 (75% average eligibility and unit 4 student characteristics). Table 4-2 shows the results obtained using data set 2 (50% average eligibility and unit 4 student characteristics).

Each table contains the final distance, the mean final distance, the number of omissions, the rank obtained by each weight in order of least mean final distance, the rank obtained by

TABLE 4-1

EFFECTS OF WEIGHTS ON DATA SET 1 FOR GROUPAL C

Weights	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Total Rank
0.5	0	7	153.720	1.450	7	14
1.0	1	3.5	173.688	1.654	6	9.5
2.0	1	3.5	185.740	1.768	4.5	8
3.0	1	3.5	185.740	1.768	4.5	8
5.0	1	3.5	187.314	1.783	3	6.5
10.0	1	3.5	188.280	1.793	1.5	5
20.0	1	3.5	188.280	1.793	1.5	5

TABLE 4-2

EFFECTS OF WEIGHTS ON DATA SET 2 FOR GROUPAL C

Weight	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Total Rank
0.5	8	6	159.209	1.624	7	13
1.0	10	1	165.834	1.727	6	7
2.0	9	3	178.225	1.837	2	5
3.0	8	6	179.457	1.831	4.5	10.5
5.0	8	6	179.457	1.831	4.5	10.5
10.0	9	3	178.225	1.837	2	5
20.0	9	3	178.225	1.837	2	5

TABLE 4-3

EFFECTIVENESS OF WEIGHTS FOR GROUPAL C - FINAL RANKS

Weights	Combined Ranks On Omissions	Combined Ranks on Mean Final Distance	Combined Total Ranks
0.5	13	14	27
1.0	4.5	12	16.5
2.0	6.5	6.5	13
3.0	9.5	9	18.5
5.0	9.5	7.5	17
10.0	6.5	3.5	10
20.0	6.5	3.5	10

TABLE 4-4

EFFECTS OF WEIGHTS ON DATA SET 1 FOR GROUPAL D

Weights	Omissions	Final Ranks	Mean Final Distance	Distance	Total Ranks	Ranks
0.5	1	6	143.790	1.369	7	13
1.0	1	6	179.908	1.720	6	12
2.0	1	6	185.267	1.764	5	11
3.0	0	2.5	188.158	1.775	4	6.5
5.0	0	2.5	192.896	1.819	1	3.5
10.0	0	2.5	192.458	1.815	2	4.5
20.0	0	2.5	192.059	1.811	3	5.5

TABLE 4-5

EFFECTS OF WEIGHTS ON DATA SET 2 FOR GROUPAL D

Weights	Omissions	Ranks	Final Distance	Mean Final Distance	Ranks	Total Ranks
0.5	18	1	127.252	1.446	7	8
1.0	8	3.5	159.974	1.632	6	9.5
2.0	7	5	175.421	1.771	5	10
3.0	13	2	166.375	1.788	4	6
5.0	8	3.5	175.879	1.794	3	6.5
10.0	6	6	181.067	1.810	1	7
20.0	5	7	181.412	1.796	2	9

TABLE 4-6

EFFECTIVENESS OF WEIGHTS FOR GROUPAL D - FINAL RANKS

Weights	Combined Ranks On Omissions	Combined Ranks on Mean Final Distance	Combined Total Ranks
0.5	7	14	21
1.0	9.5	12	21.5
2.0	11	10	21
3.0	4.5	8	12.5
5.0	6	4	10
10.0	8.5	3	11.5
20.0	9.5	5	14.5

each weight on the basis of least omissions and the total ranks on both criteria. The final distance was found by summing the weighted distances of each group member from the centroid of each final group. The omissions comprised those students not eligible for group membership because of their inability to meet the various constraints. The mean final distance was found by dividing the total final distance by the number of students placed into groups. The total ranks were found by summing the ranks obtained on each of the two criteria. In all cases, the highest rank, (7), corresponded to the most effective weighting on the criterion considered.

Table 4-3 shows that a weight of 0.5 applied in Groupal C had a clear advantage over all other weights in terms of its consistency in yielding groups with comparatively high measures of overall homogeneity (low distance) and the least number of omissions. For both data sets of high and medium eligibilities a weight of 0.5 produced the most homogeneous groups and no other weight resulted in less omissions. The weights of 0.5 applied to all skill elgibilities when these were included in the expanded list of student characteristics and was designed to reflect the importance of placing students with similar patterns of eligibilities into the same group.

Generally, the larger the weight the greater the mean distance (less homogeneity), or the smaller the weight used, the greater the overall homogeneity amongst students. The rate of increase in mean distance, however became less as the size of the weight increased. The number of omissions showed little difference among the various weights. However, the comparatively high ranks for weight 0.5 on this criterion may be inflated. For example, weight 1.0 which produced the most omissions (1 + 10) nevertheless produced only three more omissions than did a weight of 0.5 over both data sets involving 212 students. Despite this difficulty in the use of ranks, the weight 0.5 produced the least omissions over all tests.

As a consequence of its more effective performance on both criteria and over both data sets, it appears that of the weights tested, a weight of 0.5 most consistently and most effectively provided the highest degree of homogeneity as well as the least omissions. This observation led to the following recommendation:

Recommendation 1. A weight of 0.5 should be applied to skill eligibilities when using Groupal C in future tests. It may be noted that this recommendation would be unaltered by the use of either of the criteria separately as shown by the ranks of Table 4-3.

Table 4-6, which refers to the relative effectiveness of the various weights used with Groupal D, does not reveal any one weight as being clearly the most effective. This lack of a clearly superior weight is a result of two opposite trends in the two criteria - mean distance and number of omissions.

For both data set 1 (Table 4-4) and data set 2 (Table 4-5) there was a trend for an increase in weight to correspond to an increase in mean distance (a decrease in homogeneity). However, for both data set 1 and data set 2 there was a trend for an increase in weight to correspond to a decrease in omissions. This latter trend was not strong because of the generally minimal omissions for all weights for data 1 (Table 4-4) and because of the inconsistencies in the omissions for data 2 (Table 4-5). Nevertheless this weak trend in omissions and a stronger but opposite trend is homogeneity caused the near equalizing of the combined ranks for the weights 0.5, 1.0 and 2.0.

As was the case for Groupal C, the rate of increase in mean distance became less as the weights increased. No similar trend in rate of decrease was apparent in the number of omissions. On the basis of these observations the following recommendation was made.

Recommendation 2: A weight of 1.0 should be applied to all skills eligibilities when using Groupal D in all future tests.
On the basis of the 14 tests made on two sets of data, a weight of 1.0 applied in Groupal D appeared to produce a comparatively minimal number of omissions while at the same time producing a medium degree of homogeneity. The alternatives of using a weight of 0.5 provided a greater degree of homogeneity. However, the risk of incurring greater omissions was also higher.

The other alternative of using a weight of 2.0 appeared to provide much less homogeneity and about the same number of omissions. Neither alternative could be considered more acceptable than that of using a weighting of 1.0. It may also be noted from Table 4-6 that a weight of 2.0 would have been selected if the criterion for selection had been omissions and that a weight of 0.5 would have been selected on the criterion of mean distance. It is noted that the selection of a weight of 1.0 was a compromise, in that this weight ranked second highest on both criteria. This consistency in ranks was not shared by the other two alternatives considered above.

It is recognized that a more comprehensive testing program may have yielded a clearer indication of the effects of different weights. A more detailed analysis of the effects of weights is provided later in this chapter (page 193).

Selection of the Most Effective Algorithm

The purpose of this set of tests was to determine which of the four algorithms Groupal A, Groupal B, Groupal C (0.5) or Groupal D (1.0) was the most effective in terms of most consistently producing the most homogeneous groups and the least number of omitted students. Tables 4-8 to 4-24 detail the results of 16 tests performed on each of the four algorithms.

In each instance the one set of 106 students was used. However, this was the only constant element throughout the progam, all other elements (data sets, single/multiple usage, number of groups and sizes of groups) were varied. The testing program for the selection of the most effective algorithm was designed to be comprehensive in its attempt to utilize a variety of grouping situations. The chosen mix of elements was intended to provide a testing program which included a broad range of possible grouping situations thought to be typical of an instructional unit's requirements.

The frequencies with which these elements were used in the testing program are shown in Table 4-7. The testing program comprised 64 tests.

TABLE 4-7

USE OF VARIABLE ELEMENTS IN TESTING PROGRAM

Element	Frequency Of Use	Percentage Of Use
Data Set 1 (75% elig., unit 4)	16	25%
Data Set 2 (50% elig., unit 4)	8	12½%
Data Set 3 (50% elig., extremes)	4	6¼%
Data Set 4 (75% elig., extremes)	4	6¼%
Data Set 5 (33% elig., unit 4)	16	25%
Data Set 6 (75% elig., simulated)	8	12½%
Data Set 7 (75% elig., simulated)	8	12½%
Single Usage of Skills	20	31%
Multiple Usage of Skills	44	69%
5 groups of sizes 25-30, 25-30, 25-30 15-20, 5-10	48	75%
5 groups with no size constraints	4	6¼%
5 groups with exact sizes	4	6¼%
3 groups of sizes 40-50, 30-40, 20-30	4	6¼%
8 groups of sizes 22-27, 20-25, 17-22, 15-20, 10-15, 8-13, 5-10, 1-5	4	6¼%

It is evident that the testing program emphasized the multiple usage of skills option as well as the selection of five groups of sizes 25-30, 25-30, 25-30, 15-20, 5-10. The data sets most extensively used were of two types - that possessing a high degree of eligibility (data set 1) and that possessing a low degree of eligibility (data set 5). It was considered that these emphases were more representative of a typical instructional situation than were the other less frequently used elements. The results of the 64 tests are reported in categories according to the data set used. The first 16 tests involved data set 1.

Data Set 1

Tables 4-8 to 4-12 each refer to four different tests of the algorithms. All 16 tests involved data set 1 (75% average, eligibility and student characteristics from unit four of the cooperating school). As was the case in the selection of weights, the algorithms were ranked on each criterion (number of omissions and homogeneity), these ranks being of equal weight. In all cases, the highest rank 4, corresponded to the most effective algorithm. The separate ranks were then summed to yield a combined total rank.

In all tests, weights of 0.5 and 1.0 were used for Groupals C and D respectively.

Table 4-12 reveals that Groupal A most consistently produced the least omissions over the four tests on data set 1 (75% eligibility). The exception to this occurred in the case where no size constraints were applied (Table 4-10) and algorithms B and D yielded no omissions.

This position relative to A and B was reversed when exact size constraints were applied, Groupal B yielding 9 omissions compared to none from Groupal A. Over the four tests Groupal C provided only one more omission than did Groupal A. Groupal D consistently provided the most omissions, except when size constraints were not applied, in which case Groupal B yielded 9 omissions. This latter inconsistency was examined in more detail when the effects of group sizes were considered.

Groupals A, B and C consistently provided more homogeneous groupings (lesser mean final distances) than did Groupal D which in all cases provided the least homogeneous groups. Its mean final distance was on the average 27 units of distance greater than the next least effective algorithm. For data set 1, Groupal D clearly did not provide homogeneous groupings to the same extent as did the other algorithms. Of Groupals A, B, and C, B consistently provided the greatest measure of homogeneity. In no instance did Groupal C provide for more homogeneity than did Groupal B. Groupal A exceeded the homogeneity provided by Groupal B once and then only marginally (a difference of .004).

It therefore appeared that on data set 1 (75% eligibility), Groupals A and B were the most effective, with Groupal A providing less omissions and Groupal B providing greater homogeneity.

TABLE 4-8

TEST OF ALGORITHMS WITH DATA SET 1, SINGLE USAGE AND FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Total Rank
A	0	3	152.032	1.434	3	6
B	0	3	148.319	1.399	4	7
C(0.5)	0	3	153.721	1.450	2	5
D(1.0)	5	1	175.136	1.751	1	2

TABLE 4-9

TEST OF ALGORITHMS WITH DATA SET 1, MULTIPLE USAGE AND FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Total Rank
A	0	3	152.032	1.434	3	6
B	0	3	148.319	1.399	4	7
C(0.5)	0	3	153.721	1.450	2	5
D(1.0)	2	1	179.908	1.729	1	2

TABLE 4-10

TEST OF ALGORITHMS WITH DATA SET 1, MULTIPLE USAGE, FIVE GROUPS AND NO SIZE CONSTRAINTS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	3	1.5	141.239	1.371	4	5.5
B	0	3.5	145.843	1.375	3	6.5
C(0.5)	3	1.5	144.662	1.404	2	3.5
D(1.0)	0	3.5	181.537	1.712	1	4.5

TABLE 4-11

TEST OF ALGORITHMS WITH DATA SET 1, MULTIPLE USAGE, FIVE GROUPS AND EXACT SIZES

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	0	4	150.780	1.422	2	6
B	9	1	126.879	1.308	4	5
C(0.5)	1	3	145.356	1.384	3	6
D(1.0)	2	2	173.353	1.666	1	3

TABLE 4-12

ALL TESTS OF ALGORITHMS FOR DATA SET 1

Algorithms	Combined Ranks On Omissions	Combined Ranks On Mean Final Distance	Combined Total Rankings
A	11.5	12	23.5
B	10.5	15	25.5
C(0.5)	10.5	9	19.5
D(1.0)	7.5	4	11.5

TABLE 4-13

TEST OF ALGORITHMS WITH DATA SET 2, SINGLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	8	2	161.960	1.652	1	3
B	7	4	157.030	1.586	4	8
C(0.5)	8	2	159.209	1.624	3	5
D(1.0)	8	2	159.974	1.632	2	4

Data Set 2

Tables 4-13 to 4-15 refer to 8 tests conducted on Data Set 2 (50% eligibility). Two different testing situations were created by alternating single usage of skills (Table 4-13) and multiple usage of skills (Table 4-14). The results on the two sets of tests are markedly similar. It appears that the multiple usage option had no effect in the cases of Groupals A, C, and D which all produced the same numbers of omissions and the same degrees of homogeneity for both single and multiple usage of skills. The numbers of omissions yielded by all algorithms were very similar (B gave one less overall tests) and were all ranked similarily. Groupal B yielded more omissions when the multiple usage option was selected.

Consistent measures of homogeneity were produced by the different algorithms over both single and multiple usage, with Groupal B consistently yielding the most homogeneous groupings. This was also the case for data set 1, which contained higher percentages of students eligible for skills.

The failure of the multiple usage option to assign the same skills to different groups occurred because none of the eligibility remainders were greater than the eligibilities for skills not yet assigned.

From this analysis it is clear that Groupal B was the most effective algorithm. Groupal A provided the least homogeneous groups over these tests on data set 2 and was comparatively the least effective algorithm.

TABLE 4-14

TEST OF ALGORITHMS WITH DATA SET 2, MULTIPLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	8	3	161.960	1.652	1	4
B	10	1	148.786	1.549	4	5
C(0.5)	8	3	159.209	1.624	3	6
D(1.0)	8	3	159.974	1.632	2	5

TABLE 4-15

ALL TESTS OF ALGORITHMS WITH DATA SET 2

Algorithm	Combined Ranks Omissions	Combined Ranks Mean Final Distance	Combined Total Rankings
A	5	2	7
B	5	8	13
C(0.5)	5	6	11
D(1.0)	5	4	9

Data Set 3

Tables 4-17 and 4-18 refer to 8 tests performed on data containing two opposite and extreme student records.

Data Set 3 comprised Data Set 2 (50% eligibility) with two new student records substituted for the first two students in alphabetical order. The record profiles of the four students concerned are shown in Table 4-16.

TABLE 4-16

SCORE PROFILES OF EXTREME STUDENTS IN DATA SETS 3 AND 4

	Visual Language	Auditory Language	Expressive Oral	Expressive Written
Substitute 1	10	10	10	10
Replaced Student	26	28	28	38
Substitute 2	40	40	40	40
Replaced Student	20	30	34	18
Mean overall 106 Students	27.26	29.77	27.08	28.43
Standard Deviation Over All 106 Students	6.78	5.12	5.70	5.99

Data Set 4 comprised Data set 1 (75% eligibility) with the same two student substitutions.

Tables 4-17 and 4-18 show two different trends. For data set 3 (50% eligibility, extreme scores) Groupals C and D performed better on both criteria than did Groupals A and B. This position was somewhat reversed in the case of data set 4 (75% eligibility, extreme scores) where Groupal D produced 12 omissions and also the least homogeneous groups.

The effect of the extreme scores was hardly felt by Groupals A and B, both yielding one more omission, but also because of the extreme scores yielded more homogeneous groups. Groupals C and D (in particular) produced less omissions and also more homogeneous groups in the presence of extreme scores in data set 2. Groupals A and C had the effect of omitting one of the students with extreme scores from the grouping because of group size constraints.

For data with 75% eligibility the effect of extreme scores was mostly felt by Groupal D but in the reverse direction to that for data with 50% eligibility. Groupals A, B and C gave consistent results for data with and without extreme scores, but with Groupals B and C yielding more omissions with extreme scores. Groupal D produced the greatest number of omissions over both data sets with a very large increase in omissions for data set 4 (75% eligibility, extremes)

In no instance involving data set 4 were the students with extreme scores omitted from the groupings. Because of the reverse trends in both number of omissions and degree of homogeneity over both data sets 3 and 4 no one algorithm was clearly the most effective on data containing extreme scores, although Groupal C performed the most consistently for both criteria and in fact achieved the highest or equal highest ranks on both criteria (Table 4-19).

TABLE 4-17

TEST OF ALGORITHMS WITH DATA SET 3, MULTIPLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	9	1.5	152.130	1.568	2	3.5
B	9	1.5	152.251	1.569	1	2.5
C(0.5)	4	3	153.593	1.567	3	6
D(1.0)	2	4	159.499	1.533	4	8

TABLE 4-18

TEST OF ALGORITHMSWITH DATA SET 4, MULTIPLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	0	4	149.663	1.411	3	7
B	2	2	137.089	1.318	4	6
C(0.5)	1	3	152.198	1.449	2	5
D(1.0)	12	1	157.033	1.670	1	2

TABLE 4-19

ALL TESTS OF ALGORITHMS ON DATA SETS 3 AND 4

Algorithm	Combined Ranks Omission	Combined Ranks Final Distance	Combined Total Ranks
A	5.5	5	10.5
B	3.5	5	8.5
C	6	5	11
D	5	5	10

Data Set 5

Tables 4-20 to 4-24 refer to 16 tests performed on Data Set 5 (33% eligibility). This data set may be more representative of grouping situations than either data set 1 (75% eligibility) or data set 2 (50% eligibility) in as much as low eligibilities over a range of skills are frequently encountered in individualized instructional programs.

Tables 4-20 to 4-23 show that over all four sets of tests on data set 5, Groupal A gave the least omissions, 65, compared to 86 from Groupal B, 67 from Groupal C and 100 from Groupal D. Clearly, for both single and multiple usage, for different numbers and for different sizes of groups, Groupals A and C (C is A modified to include skills eligibilities as student characteristics) gave the least omissions with Groupal A slightly more effective.

There appeared to be no clear trends in the homogeneity of groups generated using data set 5. The large number of omissions produced by all procedures, but especially by Groupal D, may cause the mean final distances to be less valid comparisons of homogeneity than in other tests. Because of the large numbers of students omitted the set of students placed into groups was much smaller and possibly with different characteristics than the original set of 106 students. This doubt about the validity of mean final distances as comparable measures of homogeneity applied particularly to Groupal B and D results and to a lesser extent to the results obtained from Groupals A and C, which algorithms together produced 27% less omissions then did Groupals B and D.

The mean final distances for all algorithms were very similar for each of the four tests. In particular, Groupals A, B, and C very consistently produced similar distances (similar degrees of homogeneity). Groupal D performed more erratically, clearly yielding more homogeneous and smaller groups for five groups and multiple usage (Table 4-21) but clearly less homogeneous groups when eight groups and multiple usage were requested (Table 4-22).

Consideration of the 16 tests (4 types) on data set 5 (33% eligibility, unit 4 characteristics), led to the conclusion that Groupal A consistently yielded less omissions and that Groupals B and D produced considerably more. However these two latter algorithms produced the more homogeneous groupings. Because these groups were also smaller and because of the consequent doubt about the validity and reliability of the mean final distance as a comparable measure of homogeneity, no conclusion can be drawn about the effectiveness of the algorithms in producing homogeneous groups when the percentages of eligibilities are small.

TABLE 4-20

TESTS OF ALGORITHMS WITH DATA SET 5, SINGLE USAGE, 5 GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	15	4	152.241	1.672	3	7
B	16	2.5	158.206	1.757	1	3.5
C(0.5)	16	2.5	150.620	1.673	2	4.5
D(1.0)	19	1	145.393	1.671	4	5

TABLE 4-21

TESTS OF ALGORITHMS WITH DATA SET 5, MULTIPLE USAGE, 5 GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	15	4	152.241	1.672	2	6
B	19	2	142.688	1.640	3	5
C(0.5)	16	3	150.620	1.673	1	4
D(1.0)	32	1	109.891	1.485	4	5

TABLE 4-22

TESTS OF ALGORITHMS WITH DATA SET 5, MULTIPLE USAGE, 8 GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	10	3.5	142.641	1.485	4	7.5
B	13	2	138.280	1.486	3	5
C(0.5)	10	3.5	147.357	1.534	2	5.5
D(1.0)	15	1	148.088	1.627	1	2

TABLE 4-23

TESTS OF ALGORITHMS WITH DATA SET 5, MULTIPLE USAGE, 3 GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	25	3.5	144.806	1.787	1	4.5
B	32	2	128.546	1.737	4	6
C(0.5)	25	3.5	144.635	1.785	2	5.5
D(1.0)	34	1	128.086	1.778	3	4

TABLE 4-24

ALL TESTS OF ALGORITHMS FOR DATA SET 5

Algorithm	Combined Ranks Omissions	Combined Ranks Mean Final Distance	Combined Total Ranks
A	15	10	25
B	8.5	11	19.5
C(0.5)	12.5	7	19.5
D(1.0)	4	12	16

Data Sets 6 and 7

Tables 4-25 and 4-27 concern data set 6 which has the same eligibility data (75% eligibility) as data set 1 but with simulated student characteristics. These simulated data had the same means and standard deviations as in data set 1.

In both cases of single usage (Table 4-25) and multiple usage (Table 4-26) Groupal A yielded no omissions and Groupal C yielded one. Related algorithms B and D acted less consistently for single usage and multiple usage, both yielding more omissions in the case of multiple usage.

Considering the criterion of homogeneity, Groupal A produced the most homogeneous groups, although all four algorithms produced groups with very similar measures of homogeneity. Over both criteria Groupal A appeared to be the most effective algorithm.

Data set 7 was also similar to data set 1, having the same eligibilities but with different simulated data on student characteristics. Similar conclusions to those made on the basis of data set 6 were reached on the basis of tests on data set 7. Groupal A was the most effective algorithm, although not as clearly as previously. All algorithms yielded similar numbers of omissions. However, Groupal B yielded no omissions, a marked contrast to the case for data set 6 (Table 4-26). Groupals A, B and C also yielded very similar measures of homogeneity with Groupal A producing very slightly more homogeneous groups. Groupal D clearly and consistently yielded the least homogeneous groups.

Over both data sets 6 and 7 (both 75% eligibility and with simulated student characteristics) Groupals A and B consistently yielded the least omissions although Groupal B performed strangely for multiple usage on data set 6 (Table 4-26). Groupal A also clearly yielded the most homogeneous groups over all tests. These conclusions are consistent with the results displayed in Table 4-31.

TABLE 4-25

TESTS OF ALGORITHMS WITH DATA SET 6, SINGLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	0	3	160.319	1.512	4	7
B	0	3	171.494	1.617	2	5
C(0.5)	1	1	165.279	1.574	3	4
D(1.0)	0	3	174.884	1.645	1	4

TABLE 4-26

TESTS OF ALGORITHMS WITH DATA SET 6, MULTIPLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Mean Distance	Mean Final Distance	Rank	Final Rank
A	0	4	160.319	1.512	4	8
B	5	2	158.286	1.567	3	5
C(0.5)	1	3	165.279	1.574	2	5
D(1.0)	8	1	156.408	1.596	1	2

TABLE 4-27

ALL TESTS OF ALGORITHMS FOR DATA SET 6

Algorithm	Combined Rank Omissions	Combined Mean Final Distances	Combined Total Rankings
A	7	8	15
B	5	5	10
C(0.5)	4	5	9
D(1.0)	4	2	6

TABLE 4-28

TESTS OF ALGORITHMS WITH DATA SET 7, SINGLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	1	2.5	149.809	1.426	4	6.5
B	0	4	158.945	1.499	2	6
C(0.5)	1	2.5	156.458	1.490	3	5.5
D(1.0)	2	1	179.596	1.726	1	2

TABLE 4-29

TESTSOF ALGORITHMS WITH DATA SET 7, MULTIPLE USAGE, FIVE GROUPS

Algorithm	Omissions	Rank	Final Distance	Mean Final Distance	Rank	Final Rank
A	1	1.5	149.809	1.426	4	5.5
B	0	3.5	158.420	1.494	2	5.5
C(0.5)	1	1.5	156.458	1.490	3	4.5
D(1.0)	1	3.5	179.282	1.691	1	4.5

TABLE 4-30

ALL TESTS OF ALGORITHM FOR DATA SET 7

Algorithm	Combined Ranks Omissions	Combined Ranks Were Final Distance	Combined Total Rankings
A	4	8	12
B	7.5	4	11.5
C(0.5)	4	6	10
D(1.0)	4.5	2	6.5

TABLE 4-31

ALL TESTS OF ALGORITHMS FOR DATA SETS 6 AND 7

Algorithm	Combined Ranks Omissions	Combined Ranks Mean Final Distance	Combined Total Ranks
A	11	16	27
B	12.5	9	21.5
C(0.5)	8	11	19
D(1.0)	8.5	4	12.5

TABLE 4-32

ALL TESTS OF ALL ALGORITHMS ON ALL DATA SETS

Algorithm	Combined Ranks Omissions	Combined Ranks Mean Final Distance	Combined Total Ranks
A	48	45	93
B	40	48	88
C(0.5)	42	38	80
D(1.0)	30	29	59

Table 4-32 shows the total ranks for each of the four algorithms over all seven data sets. In all, 16 different tests were applied to each algorithm. The total ranks on each criterion-number of omissions and degree of homogeneity are shown separately together with the combined total ranks over both criteria.

Over all tests Groupais A and B most consistently yielded the most homogeneous groups with Groupal B being slightly more effective more often than Groupal A. Groupal D was clearly the most ineffective algorithm in producing homogeneous groups.

With reference to the number of omissions, Groupal A was clearly more effective than Groupals B, C and D. Groupal D was again clearly the least effective.

On the basis of the total testing program consisting of 64 tests (Table 3-7), Groupal A was the most effective algorithm in most consistently yielding the least omissions. Groupal B however, most consistently yielded the most homogeneous groups. The choice of algorithm was clearly between Groupals A and B. Groupals C and D, which involved the weighting of skills eligibilities and their use as student characteristics, were clearly less effective.

Considering both criteria of equal importance Groupal A had an overall total rank of 93 compared to an overalltotal rank of 88 for Groupal B.

Groupal A appeared to most consistently provide a comparatively high measure of homogeneity and also a comparatively small number of omissions. Accordingly, the following recommendation was made.

Recommendation 3. Groupal A should be used in the later comparison with teacher generated groupings.

Before the results of this next comparison are reported, an analysis of the effects of various elements is provided. The effects of elements such as different weights, single and multiple usage, different size constraints and different numbers of groups were considered generally when making recommendations 1, 2, and 3. A more detailed analysis of these effects is provided in the next section.

Variable Grouping Parameters and Their Effects on Groupings

The tests referred to in the previous two sections had as their primary purpose the selection of the most effective algorithm. The purpose of the evaluation provided in this section was to manipulate the various grouping parameters and to examine their effects on the groups formed. The results of this analysis were used in later recommendations for modifying the selected grouping algorithm.

Effects of Weights Applied in Groupals C and D

In an attempt to detect the effects of different weights when these are applied to skills eligibilities, eight tests were prepared on both Groupals C and D. The results of these tests are presented in Tables 4-33 and 4-34 (Groupal C) and Tables 4-35 and 4-36 (Groupal D). Each test involved data set 2 (50% eligibility, Unit 4 student characteristics), multiple usage of skills and five groups of sizes (25-30), (25-30), (25-30), (15-20), (5-10). The seven weights used previously were again used together with a weight 0.0 which made

TABLE 4-33

EFFECTS OF WEIGHTS ON GROUPAL C - DATA SET 2, MULTIPLE USAGE, FIVE GROUPS

Weight	Omissions Due to		Distance after 1st Iteration		Number of Iterations	Distance after 1st Iteration	Distance after Constraints Applied	Mean Distance after Constraints Applied
	Ineligibility	Size Constraints						
0.0	5	3	181.093	1.792	9	157.037	161.960	1.652
0.5	5	3	181.464	1.796	10	157.455	159.209	1.624
1.0	5	5	182.538	1.807	8	168.531	165.834	1.727
2.0	5	4	185.553	1.837	6	182.188	178.225	1.837
3.0	5	3	185.817	1.839	5	183.791	179.457	1.831
5.0	5	3	186.540	1.846	5	183.791	179.457	1.831
10.0	5	4	187.199	1.853	5	184.059	178.225	1.837
20.0	5	4	187.199	1.853	5	184.059	178.225	1.837

TABLE 4-34

GROUP PROFILES RESULTING FROM WEIGHTS ON GROUPAL C, DATA SET 2, MULTIPLE USAGE, FIVE GROUPS

Weight	Skills Chosen	Sizes of Final Group	Group 1 VL AL EO EW	Group 2 VL AL EO EW	Group 3 VL AL EO EW	Group 4 VL AL EO EW	Group 5 VL AL EO EW
0.0	5, 2, 1, 4, 3	30, 20, 23, 15, 10	-.21, -.26, -.02, -.12	.19, .45, -.79, -.49	.10, -.52, -.09, .04	-.01, .46, .31, .97	.61, -.07, .73, -.34
0.5	5, 2, 1, 4, 3	30, 21, 22, 15, 10	-.10, -.33, -.13, -.16	.19, .60, -.65, -.61	-.03, -.65, -.36, .03	-.07, .62, .35, 1.0	.58, -.23, .55, .13
1.0	5, 2, 1, 4, 3	30, 20, 21, 15, 10	-.36, -.27, -.01, -.16	.01, -.13, -.16, -.72	.15, -.16, -.16, .07	-.03, .49, .35, .96	.48, .24, .20, .39
2.0	5, 2, 1, 4, 3	30, 20, 22, 15, 10	-.29, p.19, -.06, -.05	.15, -.11, -.39, -.14	-.17, -.01, -.12, .29	.33, -.14, .35, -.12,	.29, .55, -.01, -.07
3.0	5, 2, 1, 4, 3	30, 21, 22, 15, 10	-.31, -.23, -.05, -.06	.18, -.04, -.36, -.12	-.17, -.01, -.12, .29	.33, -.14, .35, -.12	.29, .55, .06, .03
5.0	5, 2, 1, 4, 3	30, 21, 22, 15, 10	-.31, -.23, -.05, -.06	.18, -.09, -.36, -.12	-.17, -.01, -.12, 29	.33, -.14, .35, -.12	.29, .59, .06, .03
10.0	5, 2, 1, 4, 3	30, 20, 22 15, 10	-.29, -.19, -.06, -.05	.15, -.11, -.39, -.14	-.17, -.01, -.12, .29	.33, -.14, .35, -.12	.29, .59, -.01, -.07
20.0	5, 2, 1, 4, 3	30, 20, 22, 15, 10	-.29, -.19, -.06, -.05	.15, -.11, -.39, -.14	-.19, -.01, -.12, .29	.33, -.14, .35, -.12	.29, .55, -.01, -.07

VL = Usual Language, AL = Auditory Language, EO = Expressive Oral, EW = Expressive Written

Groupal C equivalent to Groupal A and Groupal D equivalent to Groupal B.

Table 4-33 reveals 8 to 10 omissions for each weight. Of those students omitted, four were ineligible for any of the skills initially selected by the user and all five were ineligible for the skills selected by Groupal C. When multiple usage of skills was requested, skills were selected in the order of greatest eligibility allowing for remainders of skills eligibilities to be considered in subsequent assignments of skills. Consequently, in the groupings shown in Table 4-34 the smallest group, 5, was always overloaded and most relocations originated from this smallest group when size constraints were applied. Students were then relocated in groups not exceeding their upper size limits.

Though the numbers of students omitted were the same, the students were not the same. The selection of students to be omitted (when necessary) depended upon the students' patterns of eligibilities and distances, from the centroid of the group of which they were members. The centroids of groups were always different from grouping to grouping and iteration to iteration. The imposition of size constraints and hence the removal of students from groups involved the selection of the most distant student from the overloaded group to be placed into another available group. If none were available, the student was placed into the omissions group in preference to selecting the second most distant student to undergo the same procedure. This process had the effect of increasing the number of omissions presented to the user who then makes the decision as to re-inclusion after examination of the omissions group and the accompanying diagnostic, which provides information on other eligibilities.

There was no observable relationship between the numbers of students omitted and the sizes of the weights applied in Groupal C, the number of omissions remaining fairly constant as the weight increased. Total distance after the first and after the last iteration were calculated for only those students initially placed into groups (101 students). Total distance after the imposition of size constraints excluded these other omitted students.

In all cases, the average final distance was least for a weight of 0.0 and increased with increasing weights. This rate of increase was very slow for weights, 2.0 and greater.

The effect of these heavier weights was made evident by considering the number of iterations required for convergence. The effect of increasing the weights assigned to skills was to decrease the number of iterations thus making for a more rapid convergence. However, the convergence was always towards a local optimum of less homogeneity as the weighting increased. The heavier the weighted skill eligibilities, the more influence they had in forcing students with similar patterns of eligibilities into groups for which they were eligible. Consequently students were more quickly placed into these groups.

Convergence in all tests conducted with weighted skills was always achieved within a maximum of 18 iterations. However, the convergence was not achieved as a continuous decrease in total distance. For example, the total distances at the end of each of the eight iterations required with weight 1.0 were: 182.538, 170.915, 169.904, 169.154, 169.673, 168.974, 168.531, 168.531. This phenomenon was caused by the inclusion of weighted skill eligibilities as student characteristics in the allocation-reallocation process and their exclusion in the calculation of the total distances.

No differences in skills allocated were noted (Table 4-34). This was expected because of the nature of the skills selection procedure. No skill was used more than once because remaining eligibilities were insufficient to cause the reselection of any skill. The sizes of final groups showed little change, any change being in the middle group because of ineligibilities and overloading of the smaller groups (Table 4-34).

The mean (expressed in standardized scores) of each student characteristic is shown as part of the profile of each group in Table 4-34. Negative scores can be considered to be less than the mean of the characteristic for the original group and positive scores greater than this mean. Differences in the strengths of the characteristics for each of the groups can be detected within each partition and among the partitions formed by using different weights. The influence of these weights on the distribution of the student characteristics is unclear.

The characteristics used in these groupings all apply to reading skills and when expressed in standardized score form have means of zero. As an example of the variation in the profiles of groups within the one partition consider those formed using a weight of 0.0.

(Table 4-34). The profile of each group comprised distance measures for each student on each of the student characteristics considered. The distances referred to are those of each student from the mean of each characteristic for the relevant group. The measures of the characteristics had been standardized. The groups may be characterized as below.

Characteristic	Group 1 Skill 5 30 students	Group 2 Skill 2 20 Students	Group 3 Skill 1 23 Students	Group 4 Skill 4 15 Students	Group 5 Skill 3 10 Students
Visual language	medium/ low	medium/ low	medium	medium	high
Auditory language	medium	high	low	high	medium
Expressive Oral	average	low	medium	medium/ high	high
Expressive Written	medium/ low	low	medium	high	low

Within each group of Table 4-34 there was little if any change in group profile for weights of 2.0 or greater, the most noticeable changes in profiles for each group occurring for weights of 0.0, 0.5 and 1.0. The most significant of these differences occurred in the smaller groups, 4 and 5.

Tables 4-35 and 4-36 refer to the effects of weights on Groupal D. In Groupal D, groups are initially formed free of constraints and then skills are assigned to these groups on the basis of greatest eligibility within each group or if the multiple usage

TABLE 4-35

EFFECTS OF WEIGHTS ON GROUPAL D - DATA SET 2, MULTIPLE USAGE, FIVE GROUPS

	Omissions Due to							
Weight	Ineligibility	Size Constraints	Distance after 1st Iteration		Number of Iterations	Distance after 1st Iteration	Distance after Constraints applied	Mean Distance After Constraints applied
0.0	7	3	151.449	1.428	18	137.940	148.786	1.549
0.5	10	8	156.096	1.472	11	137.421	127.252	1.446
1.0	5	3	165.814	1.564	12	156.792	159.974	1.632
2.0	7	0	195.033	1.839	6	191.908	175.421	1.771
3.0	12	1	195.291	1.842	10	190.663	166.375	1.788
5.0	8	0	195.575	1.845	5	192.025	175.879	1.794
10.0	6	0	195.257	1.842	4	193.466	181.067	1.810
20.0	5	0	193.175	1.822	7	191.874	181.412	1.796

TABLE 4-36

GROUP PROFILES RESULTING FROM WEIGHTS ON GRPUAL D, DATA SET 2, MULTIPLE USAGE, FIVE GROUPS

Weight	Skills Chosen	Size Final Group	Group 1 VL	Group 1 AL	Group 1 ED	Group 1 EW	Group 2 VL	Group 2 AL	Group 2 ED	Group 2 EW	Group 3 VL	Group 3 AL	Group 3 ED	Group 3 EW	Group 4 VL	Group 4 AL	Group 4 ED	Group 4 EW	Group 5 VL	Group 5 AL	Group 5 ED	Group 5 EW
0.0	4, 2, 5, 5, 1	24, 22, 25, 15, 10	.19,	.50,	.40,	.73	.52,	-.63,	-.07,	-.19	-.02,	.42,	.35,	-.11	-.84,	-.48,	-.93,	-.58	-.57,	-.78,	-1.09	-.34
0.5	5, 2, 5, 6, 3	18, 25, 20, 15, 10	-1.07,	-.39,	-1.13,	-.55	.26,	-.58,	-.28,	-.25	.08,	.30,	.50,	-.39	.03,	.49,	.17,	.97	.82,	.75,	.83,	1.00
1.0	1, 2, 6, 3, 5	25, 27, 21, 15, 10	-.43,	-.49,	-.52,	-.37	.12,	-.27,	-.16,	.01	.09,	.53,	.07,	.82	.38,	.33,	.52,	-.76	.46,	.36,	.45,	.59
2.0	1, 5, 6, 3, 3,	20, 28, 26, 15, 10	-.28,	-.07,	-.18,	.38	-.49,	-.32,	-.03,	-.17	.28,	.07,	-.41,	.00	.34,	.09,	.42,	-.72	.79,	.55,	.34,	.16
3.0	5, 1, 5, 61	24, 30, 16, 13, 10	.03,	-.07,	-.05,	.19	-.08,	-.05,	-.22,	-.31	-.17,	-.18,	.15,	-.09	.40,	.22,	-.50,	-.05	-.39,	-.03,	.45,	.90
5.0	2, 1, 5, 2, 6	26, 30, 21, 12, 9	.15,	-.15,	-.06,	.15	-.01,	-.29,	-.23,	-.21	-.10,	-.01,	-.03,	-.15	-.14,	.40,	-.33,	-.27	-.12,	.61,	.44	1.08
10.0	3, 1, 5, 2, 1	26, 27, 25, 12, 10	.32,	.27,	.10,	.08	.09,	-.16,	-.17,	-.17	-.21,	-.36,	-.07,	-.11	-.09,	.30,	-.30,	-.24	-.45,	-.07,	.24,	.76
20.0	3, 5, 4, 6, 1	27, 30, 20, 14, 10	.37,	-.06,	.19,	-.08	-.28,	.04,	.04,	-.04	-.08,	-.37,	-.37,	-.26	.17,	.18,	-.53,	-.19	-.25,	.28,	.45,	.83

option is used, remaining eligibilities are also considered in subsequent assignments of skills. As observed earlier, (Table 4-5) the total number of omissions tends to decrease as the weight increases. However, this is not the case for omissions caused by ineligibility, these omissions being dependent on the skills assigned and the individual student's ineligibilities. The criterion for assignment of skills to groups only considered eligibilities over each group and assigned that skill which had the greatest eligibility in that particular group. Sizes are matched with skills in the order of greatest eligibility with greatest size (based on the lower group size limit). Consequently, this assignment technique was not based on eligibility data for the whole data set as was the case for Groupal C.

The number of omissions due to size constraints definitely decreased with increasing weights perhaps reflecting the increasing influence of patterns of skills eligibilities in the initial allocations, which similarities in turn affect the assignment of skills to groups. The number of omissions due to size constraints in Groupal D was less than in Groupal C due to the fact that the smaller groups were not as overloaded in Groupal D as in Groupal C.

All mean distances (after the first iteration, after the last iteration and after constraints have been applied) increased as the weights increased. The lowest weights 0.0 and 0.5 and 1.0 provided the smaller distances. For weights of 2.0 and higher, the distances increased very slowly.

Unlike Groupal C, the same skills were often selected by Groupal D indicating a wider range in the maximum eligibilities for the different groups as compared with eligibilities by all students for each skill. When requested, multiple use of skills seemed to be achieved more often by Groupal D than by Groupal C.

The same variation in the profiles of groups within the one partition was evident in the groups produced by Groupal D as was in the case with Groupal C.

The groups formed by Groupal D with a weight of 0.0 (Table 4-35) may be characterized as below.

	Group 1	Group 2	Group 3	Group 4	Group 5
Characteristic	Skill 4 24 students	Skill 2 22 students	Skill 5 25 students	Skill 5 15 students	Skill 1 10 students
Visual language	medium/ high	high	medium	low	low
Auditory language	high	low	high	low	low
Expressive Oral	high	medium	high	low	low
Expressive Written	high	medium/ low	medium	low	low

This set of profiles is dissimilar to that produced by Groupal C under the same conditions, making clear the fact that the different algorithms produce groups with different characteristics.

Effects of Single and Multiple Usage of Skills

Tables 4-37 to 4-40 refer to 6 tests designed to determine the effectiveness of both the single usage of skills and the multiple usage of skills options.

Data set 1 (75% eligibility), data set 2 (50% eligibility and data set 5 (33% eligibility) were used in this set of tests. The actual eligibilities for the six skills in these data sets are shown in Table 3-5 but the ranges of these skills eligibilities were:

(i) 86-75 = 11 for data set 1,
(ii) 60-43 = 17 for data set 2,
(iii) 42-25 = 17 for data set 5.

Table 4-37 which refers to the effects of the single and multiple usage option in Groupal A clearly shows that in no instance was the multiple usage of skills affected. In those instances where the multiple usage option was selected, Groupal A always selected different skills. This result was probably a function of the somewhat uniform eligibilities for the skills considered in each of the data sets. Skills were assigned in the order of greatest eligibility and in the case of multiple usage this was modified to consider the remainder (difference between greatest eligibility and lower limit corresponding group size) of the first skill eligibility in latter assignments of skills. In the data sets used, this difference was always smaller than the other

TABLE 4-37

EFFECTS OF SINGLE/MULTIPLE USAGE ON GROUPS FORMED BY GROUPAL A

		Omissions						Group 1	Group 2	Group 3	Group 4	Group 5
		Eligi-bility	Size Con-straints	Final Distance	Mean Final Distance	Skills Chosen	Size of Groups	VL AL ED EW	VL AL ED EW	VL AL EW EW	VL AL ED EW	VL AL ED EW
Data Set 1	Single	0	0	152.032	1.434	3,6,5, 1,2	30,26, 25,15, 10	.68, .73, .62, -.19	-.88, -.41, -.88, -.59	-.48, .43, .19, .72	.15, -.97, .19, .72	1.23, -.79, -.39, .13
	Multiple	0	0	152.032	1.434	3,6,5, 1,2	30,26, 25,15, 10	.08, .73, .62, -.49	.35, -.41, -.88, -.59	-.48, .43, .19, .72	.15, -.97, .16, .13	1.23, -.79, -.29, .13
Data Set 2	Single	5	3	161.960	1.652	5,2,1, 4,3	30,20, 23,15, 10	-.21, -.26, -.02, -.12	-.19, .45, -.79, -.49	.10, -.52, -.09, .04	-.01, .46, .31, .97	.61, -.07, .73, -.54
	Muliple	5	3	161.960	1.652	5,2,1, 4,3	30,20, 23,15, 10	-.21, -.26, -.02, -.12	-.19, .45, -.79, -.49	.10, -.52, .09, .04	.04, .46, .31, .97	.61, -.07, .73, -.54
Data Set 5	Single	12	3	152.241	1.672	2,1,5, 6,4	25,18, 23,15, 10	-.45, -.47, -.60, -.49	-.02, -.32, -.07, -.18	.13, .04, .45, -.09	.09, .67, .14, .51	.76, -.58, -.47, .66
	Multiple	12	3	152.241	1.672	2,1,5, 6,4	25,18, 23,15, 10	-.45, -.17, -.60, -.49	-.02, -.32, -.07, -.18	.13, .04, .45, .09	.09, .67, .14, .51	.76, -.58, .47, .66

TABLE 4-38

EFFECTS OF SINGLE/MULTIPLE USAGE ON GROUPS FORMED BY GROUPAL B

		Omissions										
		Eligibility	Size Constraints	Final Distance	Mean Final Distance	Skills Choosen	Size of Groups	Group 1 VL AL ED EW	Group 2 VL AL ED EW	Group 3 VL AL ED EW	Group 4 VL AL ED EW	Group 5 VL AL ED EW
Data Set 1	Single	0	0	148.319	1.399	3,4,5, 6,1	30,22, 27,15,7	.42, .76, .68, .86	.68, -.81, -.45, .00	-.10, .41, -.11, -.76	-1.13, -.19, -.68, -.03	-1.62, -1.30, -1.29, -.69
	Multiple	0	0	148.289	1.396	3,4,3, 6,1	28,27, 26,15, 10	.44, .76, .74, .94	.71, -.78, -.45, .08	-.10, .48, .11, -.89	-.95, -.03, -.75, -.14	-1.46, -1.21, -.82, -.31
Data Set 2	Single	5	0	157.030	1.555	4,2,5, 1,3	23,23, 28,15, 10	.21, .47, .40, .89	.37, -.77, -.11, -.09	.08, .32, .23, -.24	-.70, -.42, -.60, -.12	-.69, .04, -1.02, -1.17
	Multiple	7	3	148.786	1.549	4,2,5, 5,1	24,22, 25,15, 10	.19, .50, .40, .73	.52, -.63, -.07, -.19	-.02, .42, .35, -.11	-.84, -.48, -.93, -.58	-.57, -.78, -1.09, -.34
Data Set 5	Single	14	2	158.206	1.757	6,1,5, 2,3	20,17, 29,15, 9	-.08, -.38, -.10, .41	.35, -.58, -.25, -.11	.74, .07, .37, -.18	-.36, -.11, -.45, -.36	-.19, -.43, -.38, -.44
	Multiple	18	1	142.688	1.650	6,1,5, 1,2	21,13, 30,21, 10	-.13, .45, -.16, .45	.81, -.36, -.24, .11	-.03, .03, .27, -.22	-.98, -.14, -.78, -.82	.32, .32, .27, .09

EFFECTS OF SINGLE/MULTIPLE USAGE ON GROUPS FORMED BY [illegible] C

		[illegible] Eligibility	Size Constraints	Final Distance	Mean Final Distance	Skills Chosen	Size of Groups	Group 1 VL AL ED [illegible]	Group 2 VL AL ED [illegible]	Group 3 VL AL ED [illegible]	Group 4 VL AL ED [illegible]	Group 5 VL AL ED [illegible]
Data Set 1	Single	0	1	173.688	1.654	3,6,5, 1,2	28,29, 24,15, 10	.73, .35, .41, .21	-.56, -.17, -.60, -.52	-.08, .42, .31, .39	-.70, -.58, -.29, -.03	.85, -.54, .37, -.07
	Multiple	0	1	173.688	1.654	3,6,5, 1,2	27,29, 24,18, 10	.73, .35, .41, .21	-.56, -.17, -.60, -.52	-.08, .42, .31, .39	-.70, -.58, -.29, -.03	.85, -.54, .37, -.09
Data Set 2	Single	5	5	165.834	1.727	5,2,1, 4,3	30,20, 21,15, 10	-.34, -.27, -.01, -.16	-.01, -.13, -.[illegible], -.72	.13, -.16, -.16, .07	-.08, .43, .35, .95	.46, .24, .20, .39
	Multiple	5	[illegible]	[illegible].834	1.7[illegible]	[illegible]	[illegible], 21,15, 10	-.[illegible], -.27, -.[illegible], -.16	-.[illegible], -.[illegible], -.60, -.72	.13, -.16, -.[illegible], .07	-.03, .40, .35, [illegible]	.46, .[illegible], .20, .39
Data 3	Single	12	2	158.869	1.7[illegible]	2,1,[illegible], 6,4	[illegible],27, 21,15, 10	-.[illegible], -.[illegible], -.36, -.27	-.[illegible], -.45, -.23, -.46	.07, .08, .[illegible], .05	.15, .[illegible], .[illegible], .35	.2[illegible], .[illegible], .09, .90
	Multiple	12	2	158.869	1.726	2,[illegible], 6,4	[illegible],27, 21,13, 10	-.[illegible], -.[illegible], -.[illegible], -.27	-.[illegible], -.53, -.[illegible], -.46	.07, .[illegible], .40, .05	.15, .44, .02, .35	.[illegible], .43, .09, .90

TABLE 4-40

EFFECTS OF SINGLE/MULTIPLE USAGE ON GROUPS FORMED BY GROUPAL D

		Omissions										
		Eligibility	Size Constraints	Final Distance	Mean Final Distance	Skills Chosen	Size of Groups	Group 1 VL AL ED EW	Group 2 VL AL ED EW	Group 3 VL AL ED EW	Group 4 VL AL ED EW	Group 5 VL AL ED EW
Data Set 1	Single	0	5	175.136	1.734	4,6,3, 1,4	22,24, 30,15, 10	-.59, -.47, -.38, -.07	-.05, .03, -.17, -.17	.53, -.05, .15, -.03	.03, .54, .68, .71	-.25, .43, .09, -.47
	Multiple	2	0	179.908	1.729	4,3,3, 3,3	21,28, 30,15, 10	-.71, -.57, -.56, -.15	.29, .06, -.13, -.17	.38, -.01, .08, -.01	-.07, .46, .70, .75	-.25, .43, .09, -.47
Data Set 2	Single	5	3	159.974	1.632	1,2,6, 3,5	25,27, 21,15, 10	-.43, -.49, -.52, -.37	.12, -.27, -.16, .01	.09, .53, .07, .82	.38, .33, .52, -.76	.46, .36, .45, .59
	Multiple	5	3	159.974	1.632	1,2,6, 3,5	25,27, 21,15, 10	-.43, -.49, -.52, -.37	.12, -.27, -.16, .01	.08, .53, .09, .82	.38, .33, .52, -.76	.46, .36, .45, .59
Data Set 5	Single	14	5	145.393	1.671	2,6,1, 5,3	24,22, 16,15, 10	-.20, -.20, -.64, -.39	.15, .40, .16, .53	.13, -.27, -.19, -.22	-.44, -.29, .37, -.23	-.88, .32, .69, 33
	Multiple	24	8	109.891	1.485	2,6,2, 5,3	16,21, 12,15, 10	-.65, -.54, -1.15, -.78	.21, .32, .18, .47	.53, .47, .37, .26	-.44, -.29, .37, -.23	.85, .40, .62, .56

skills eligibilities and hence that skill was not assigned again.

Groupal C (Groupal A modified to consider weighted skills eligibilities as student characteristics) likewise failed to assign the same skill more than once (Table 4-39). Consequently identical groups were formed for both usage options. As a result no information was obtained on the differential effects of single/multiple usage with Groupals A and C.

Groupals B and D however were able to assign the same skill more than once and did so in each of the tests on data sets 1, 2 and 5 (Table 4-38 and 4-40). That this was so, is again a function of the procedure for assigning skills in Groupals B and D and also of the data sets used. In both Groupals B and D, groups were initially formed free of constraints, after which skills were assigned on the basis of greatest eligibility within each group. When the multiple usage option was chosen, skills were assigned to any groups in which that skill had the greatest eligibility. Otherwise only eligibilities for unused skills were considered. This meant that skills were assigned on the basis of subsets of the overall groups' eligibilities. Smaller eligibilities with comparatively wide differences between skills eligibilities were noticeable. For example, for data set 2, multiple usage (Table 4-38) the skills eligibilities were as below:

Skills Groups	1	2	3	4	5	6
1	14	15	15	20	16	18
2	13	16	9	13	14	12
3	12	13	13	6	15	7
4	9	10	5	6	12	3
5	4	4	4	2	3	3

Consequently, the skills selected for groups 1, 2, 3, 4, and 5 were respectively skills 4, 2, 5, 5, and 1. Correspondingly, the skills selected with the single usage option were 4, 2, 5, 1, 3.

In none of the tests did the multiple usage option produce less omissions than the single usage option. Rather, single usage consistently yielded fewer omissions especially so because of ineligibility.

However, in all cases multiple usage did result in more homogeneous groupings but these differences were usually only minor. These same minor differences in homogeneity were reflected in similar minor differences in the profiles of each of the groups formed (Tables 4-38).

The same trends were observed for Groupal D as for Groupal B in that slightly more homogeneous groups resulted from multiple usage of skills (Table 4-40). These differences were again minor, with similar minor changes in the profiles of the groups formed.

Effects of Size Constraints

Tables 4-41 to 4-44 refer to the effects of different sets of group sizes on the groupings produced by each algorithm. In each test the multiple usage option was selected to form five groups from data set 1. The three different sets of group sizes were:

(i) 25-30, 25-30, 25-30, 10-15, 5-10 (perhaps a typical request for a unit of 106 students with five instructional staff),

(ii) 30, 25, 20, 16, 15 (exact sizes), and

(iii) 1-99, 1-99, 1-99, 1-99, 1-99 (unconstrained).

Table 4-41, which refers to Groupal A, indicates that skills were selected more than once when group sizes were unconstrained (1-99). This was possible because of the small lower limit (1) subtracted from a high skill eligibility (86), which when repeatedly subtracted gave a remainder in excess of 83, the next highest skill eligibility. This was not the case, however when sizes were expressed exactly. The sizes requested 30, 25, 20, 16, 15 did not permit any of the remainders to be greater than eligibilities and therefore skills were not selected more than once. The assignment of the same skill to four of the five groups when group sizes were unconstrained also forced three students to be omitted from the grouping because they were ineligible for either of the two skills chosen. The lower limit of each group size appeared to be a most influential factor in the assignment of skills and consequently on the effectiveness of the grouping procedure. It appeared that assignment of the same skill to the majority of groups may increase the number of omissions. This multiple assignment appears to be a by-product of having a series of very small lower limits.

Unconstraining group sizes resulted in more homogeneous groups formed by Groupal A, although any differences in homogeneity were slight, especially between similar group sizes expressed in terms of a range or expressed exactly. Similar small differences between the profiles of the groups also resulted (Table 4-41). Removing size constraints however did result in groups with profiles dissimilar to the other two sets of groupings, apparently because no relocations of students were made because of size constraints. This absence of relocations also resulted in 30 students being assigned to Group 5. In all other groupings, group 5 was the smallest group (5-10), these severe size constraints resulting in the relocation of many students.

Similar observations to those for Groupal A also applied to the groups formed by Groupal C (Table 4-43).

The same trends in the homogeneity of groups observed for Groupals A and C also applied to Groupals B and D. The relaxation of size constraints and the specification of exact sizes both resulted in slightly more homogeneous groupings than when the group sizes were specified as ranges. The specification of exact sizes with Groupals B and D resulted in more omissions because of size constraints. This was due to the lowering of the upper limit for two of the larger groups, thereby forcing relocations of students ineligible for other groups. One skill was assigned to three groups and only three different skills were assigned.

TABLE 4-41

EFFECTS OF SIZE CONSTRAINTS ON GROUPS FORMED BY GROUPAL A

Group size constraints	Omissions		Final Distance	Mean Final Distance	Skills Chosen	Sizes of Groups	Group 1	Group.2	Group 3	Group 4	Group 5
	Ineligi-bility	Size Con-straints					VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW
25-30, 25-30, 25-30, 10-15, 5-10	0	0	152.032	1.434	3,6,5, 1,2	30,26, 25,15, 10	.68, .73, .62 -.19	-.85, -.41, -.88, -.59	-.45, .43, .19, .72	.15, -.97, .16, .13	1.23, .79, -.29, .13
30, 25, 20, 16, 15 (exact)	0	0	150.780	1.422	3,6,5, 1,2	30,25, 20,16, 15	.59, .71, .57, -.27	-1.05, -.49, -.05, -.55	-.33, .61, .46, .86	.03, -1.01, .12, .12	.97, -.35, -.45, .19
1-99, 1-99 1-99, 1-99 1-99 (open)	3	0	141.239	1.371	3,3,3, 3,6	25,18, 18,12, 30	.31, .78, .65 1.05	.45, .59, .73, -1.04	1.01, -.67, -.36, .06	-1.34, -.61, -1.39, -.96	-.62, -.27, -.18, .03

TABLE 4-42

EFFECTS OF SIZE CONSTRAINTS ON GROUPS FORMED BY GROUPAL B

Group size constraints	Omissions		Final Distance	Mean Final Distance	Skills Chosen	Sizes of Groups	Group 1	Group 2	Group 3	Group 4	Group 5
	Ineligi-bility	Size Con-straints					VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW
25-30, 25-30, 25-30, 10-15, 5-10	0	0	148.280	1.398	3,4,5, 6,1	28,27,26, 15,10	.46, .76, .74, .94	.71, -.78, -.45, -.08	-.10, .48, .11, -.89	-.95, -.03, -.75, -.14	-1.46, -1.21, -.82, -.31
30, 25, 20, 16, 15 (exact)	1	8	126.879	1.308	3,3,4, 3,6	27,21,20, 14,15	.32, .71, .60, 1.06	.16, .32, .83, -.87	.77, -.99, -.49, .14	-.36, .24, -1.09, .64	-1.04, -.53, -1.08, -.72
1-99, 1-99, 1-99, 1-99, 1-99 (open)	0	0	145.843	1.375	3,4,2, 3,6	24,19,23, 22,18	.42, .78, .88, .90	1.03, -.94, -.28, -.04	-.26, -.11, -.07, .57	-.08, .56, .19, -1.00	-1.20, -.59, -1.03, -.67

TABLE 4-43

EFFECTS OF SIZE CONSTRAINTS ON GROUPS FORMED BY GROUPAL C

Group Size Constraints	Omissions		Final Distance	Mean Final Distance	Skills Chosen	Sizes of Groups	Group 1	Group 2	Group 3	Group 4	Group 5
	ineligibility	Size Constraints					VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW
25-30, 25-30, 25-30, 10-15, 5-10	0	0	153.721	1.450	3,6,5, 1,2	30,28, 23,15, 10	.59, .79, .62, -.08	-.65, -.42, -.75, -.64	-.46, .55, .27, .77	-.01, -1.18, -.28, .06	1.11, -.70, .06, .19
30, 25, 20, 16, 15 (exact)	0	1	145.356	1.384	3,6,5, 1,2	30,25, 20,16, 14	.59, .79, .62, -.08	-.94, -.46, -.90, -.66	-.33, .65, .46, .85	-.08, -1.05, -.21, .12	1.16, -.60, .04, .17
1-99, 1-99 1-99, 1-99 1-99 (open)	3	0	144.662	1.404	3,3,3, 3,6	15,18, 27,26, 17	-1.13, -.58, -1.38, -.78	.90, -.98, -.03, -.02	.10, .67, .38, -.69	.34, .65, .66, 1.08	-.66, -.35, -.31, .03

TABLE 4-44

EFFECTS OF SIZE CONSTRAINTS ON GROUPS FORMED BY GROUPAL D

Group Size Constraints	Omissions		Final Distance	Mean Final Distance	Skills Chosen	Sizes of Groups	Group 1	Group 2	Group 3	Group 4	Group 5
	Ineligibility	Size Constraints					VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW
25-30, 25-30, 25-30, 10-15, 5-10	2	0	179.908	1.729	4,6,3, 15	22,24, 30,15, 10	-.59, -.47, -.38, -.07	-.05, .03, -.32, -.07	.53, -.05, .15, -.03	.03, .54, .68, .71	-.25, .43, .09, -.47
30, 25, 20, 16, 15 (exact)	0	2	173.353	1.666	5,3,3, 2,3	24,23, 30,15, 10	.48, .68, .51, .03	.94, .46, .81, .57	-.42, -.56, -.33, -.86	-.08, -1.02, -.32, -.21	1.20, .45, .07, .33
1-99, 1-99, 1-99, 1-99, 1-99 (open)	0	0	181.537	1.712	4,5,3, 1,3	16,16, 20,27, 27	-.13, -.81, -.76, -.24	-.70, -.52, -.32, -.03	.24, .40, .13, -.06	.51, .17, .67, .30	-.20, .37, -.02, -.18

Effects of Numbers of Groups

To test the effect of different numbers of groups on the homogeneity of groups and the number of omissions from groupings formed by the four algorithms, partitions for 3 groups, 5 groups and 8 groups were requested using data set 5 and multiple usage. The results are presented in Tables 4-45 to 4-48.

Requesting different numbers of groups for a constant number of students necessitates having the groups of different sizes. The group sizes accompanying each different number of groups are shown in Tables 4-45 to 4-48. Groupals A, B, C, and D all showed a trend to yield a decreasing number of omissions as the number of requested groups increases. All algorithms share the trend of decreasing omissions due to ineligibilities. Groupals A and C consistently produced less omissions than either Groupals B and D over all numbers of groups.

Groupals A, B and C all showed a trend to produce more homogeneous groups for increasing numbers of groups. Groupal D behaved erratically producing less homogeneity for 8 groups than for 5 groups. This erratic behavior may have been a function of the larger number of omissions (32). The much reduced number of students (106-32 or 74) on which the grouping was based casts doubt on the comparability of the mean final distance as a measure of homogeneity.

The level of homogeneity over these groups was very much the same for all four algorithms. For five groups, however, Groupal A performed most effectively, as did Groupal D for eight groups. The comparative effectiveness of each of the algorithms for different numbers of groups therefore remains unclear.

TABLE 4-45

EFFECTS OF NUMBERS OF GROUPS ON GROUPINGS FORMED BY GROUPAL A

Number of Groups	Omissions		Distance	Mean Final Distance	Skills Assigned	Group Sizes
	Eligibility	Size Constraints				
3 (40-50, 30-40, 20-30)	25	0	144.806	1.787	2, 1, 5	30, 25, 26
5 (25-30 25-30 25-30 15-20 5-10)	12	3	152.241	1.672	2, 1, 5, 6, 4	25, 18, 23, 15, 10
8 (22-27 20-25 19-22 15-20 10-15 8-13 5-10 1-5)	10	0	142.641	1.485	2, 1, 5, 6, 4, 3, 4, 5	21, 12, 18, 13, 10, 11, 9, 2

TABLE 4-46

EFFECTS OF NUMBERS OF GROUPS ON GROUPINGS FORMED BY GROUPAL B

Number of Groups	Omissions		Distance	Mean Final Distance	Skills Assigned	Group Sizes
	Eligibility	Size Constraints				
3 (40-50 30-40 20-30)	32	0	128.546	1.737	6, 1, 2	21, 24, 29
5 (25-30 25-30 25-30 15-20 5-10)	18	1	142.688	1.659	6, 1, 5, 1, 2	21, 13, 30, 13, 10
8 (22-27 20-25 17-22 15-20 10-15 8-13 5-10 1-5)	12	1	138.280	1.503	6, 5, 5, 4, 1, 1, 2, 2	14, 10, 15, 18, 11, 10, 10, 5

TABLE 4-47

EFFECTS OF NUMBERS OF GROUPS ON GROUPINGS FORMED BY GROUPAL C

Number of Groups	Omissions		Distance	Mean Final Distance	Skills Assigned	Group Sizes
	Eligibility	Size Constraints				
3 (40-50 30-40 20-30)	25	0	144.635	1.785	2, 1, 5	29, 25, 27
5 (25-30 25-30 25-30 15-20 5-10)	12	4	150.620	1.673	2, 1, 5, 6, 4	22, 18, 25, 15, 10
8 (22-27 20-25 17-22 15-20 10-15 8-13 5-10 1-5)	10	0	147.357	1.534	2, 1, 5, 6, 4, 3, 4, 5	17, 11, 18, 12, 10, 13, 10, 5

TABLE 4-48

EFFECTS OF NUMBERS OF GROUPS ON GROUPINGS FORMED BY GROUPAL D

Number of Groups	Omissions		Distance	Mean Final Distance	Skills Assigned	Group Sizes
	Eligibility	Size Con straints				
3 40-50, 30-40, 20-30	34	0	128.086	1.778	1, 6, 3	30, 21, 21
5 25-30, 25-30, 25-30, 15-20, 5-10	24	8	109.891	1.485	2, 6, 2, 5, 3	16, 21, 12, 15, 10
8 22-27, 20-25, 17-22, 15-20, 10-15, 8-13, 5-10, 1-5	15	0	148.088	1.627	2, 1, 2, 4, 3, 5, 3, 3	13, 20, 20, 5, 10, 13, 8, 2

Effects of Different Methods of Selecting Seed Points

Groupals B and D both possessed an option allowing user specification of seed points, as opposed to their calculation as part of each algorithm. To test the effectiveness of the two algorithmic determinations of seed points against the alternative of user specification of seed points, a series of tests was conducted in which random selections, systematic selection and teacher selection of seed points were considered.

The systematic selection involved the selection of the first five students in alphabetical order. The teacher selection was performed by the unit 4 leader who also selected five students. Table 4-49 shows the seed points selected by the different methods.

Table 4-50 shows the results of these tests using Groupal B, and Table 4-51 shows the results for Groupal D. In each test, data set 1, five groups and multiple usage were used.

Table 4-50 shows that Groupal B was the most efficient in yielding the least omissions (zero) compared with a range of one to four omissions for random selections and two omissions for systematic selection and the teachers selection. Data set 1 comprised 75% eligibility. Groupal B also yielded the most homogeneous groups although the differences between selection methods were negligible.

The distances after the last iteration ranged from 133.675 for a random selection to 139.034 for Groupal B's selection - an improvement of 4%. These results suggest that the local optimal distance achieved by Groupal B may not be too distant from the true optimum.

TABLE 4-48

SEED POINTS SELECTED FOR FIVE GROUPS BY DIFFERENT METHODS

Selected Points / Method of Selection	1	2	3	4	5
Groupal B proportionate division	.61, .64, .78, .73	.13, .13, .27, .18	.08, -.28, -.33, -.54	-1.22, -.67, -.92, -.21	-1.61, -1.12, -1.77, -1.46
Groupal D proportionate division	-.67, -.63, -.81, -.66	.22, -.04, .03, .10	.25, .22, .31, .16	.23, .66, .72, .73	.60, .96, .81, .59
Random 1	.30, .30, .30, .30	.40, .40, .40, .40	.50, .50, .50, .50	.60, .60, .60, .60	.70, .70, .70, .70
Random 2	-.30, -.30, -.30, -.30	-.40, -.40, -.40, -.40	-.50, -.50, -.50, -.50	-.60, -.60, -.60, -.60	-.70, -.70, -.70, -.70
Random 3	-.30, -.30, -.30, -.30	.40, .40, .40, .40	-.50, -.50, -.50, -.50	.60, .60, .60, .60	-.70, -.70, -.70, -.70
First Data Units	-.19, -.35, .16, 1.6	-1.1, .04, 1.22, -1.7	.11, .43, -.54, .26	1.29, .83, -.19, .93	1.58, -.35, -.54, -.41
Teacher	1.88, -.35, .16, .26	1.0, .04, 1.22, -.1.7	-.78, .83, -.19, -.41	1.58, -1.1, -.89, .59	-1.19, 1.22, 0.51, -2.7

TABLE 4-50

EFFECTS OF DIFFERENT METHODS OF SELECTION OF SEED POINTS IN GROUPAL B

Method of Selection	Distance After 1st Iteration	Number of Iterations	Distance After Last Iteration	Number of Omissions	Final Distance	Mean Final Distance
proportionate division	151.814	11	139.034	0 + 0	148.280	1.398
Random 1	195.923	14	138.072	0 + 1	151.580	1.443
Random 2	200.780	8	136.606	0 + 2	154.222	1.482
Random 3	161.911	11	133.675	1 + 3	146.944	1.440
First data units	168.805	12	137.643	1 + 1	150.213	1.444
Teacher	186.320	11	135.309	0 + 2	147.613	1.419

TABLE 4-51

EFFECTS OF DIFFERENT METHODS OF SELECTION OF SEED POINTS IN GROUPAL D

Method of Selection	Distance After 1st Iteration	Number of Iterations	Distance After Last Iteration	Number of Omissions	Final Distance	Mean Final Distance
Proportionate division	169.792	9	178.589	2	179.908	1.729
Random 1	195.923	16	176.286	2	172.243	1.656
Random 2	200.780	12	168.834	0	175.182	1.652
Random 3	161.911	7	158.970	2	165.117	1.587
First data units	168.805	13	168.733	0	179.664	1.695
Teacher	186.320	13	161.489	2	162.789	1.565

Different results in terms of both omissions and homogeneity were achieved with the proportionate division method of Groupal D which yielded two omissions as compared with no omissions achieved from one of the random selections of seed points (Table 4-51). Also Groupal D's selection of seedpoints provided the least homogeneous groupings.

Over all tests, the selection of seed points utilizing the algorithm which was a part of Groupal B provided the least omissions and the most homogeneous groups.

Effects of Extreme Student Data

The substitution of two sets of student scores - one extremely high, the other extremely low for the first two students scores (alphabetically), had a negligible effect on the means of the student characteristics but a predictably greater effect on the variance of these scores.

TABLE 4-52

SUMMARY STATISTICS FOR DATA SETS (i) INCLUDING EXTREME SCORES AND (ii) EXCLUDING EXTREME SCORES

	Including Extreme Scores				Excluding Extreme Scores			
	VL	AL	EO	EW	VL	AL	EO	EW
Mean	27.30	29.70	26.92	28.38	27.26	29.77	27.08	28.43
Variance	49.74	30.85	36.69	38.50	45.97	26.21	32.47	35.92
St. Dev.	7.05	5.55	6.06	6.20	6.78	5.12	5.70	5.99

TABLE 4-53

EFFECTS OF EXTREME SCORES ON SEED POINTS FOR DATA SET 1

	VL	AL	EQ	EW
Data Set 1				
Group 1	.61	.64	.78	.73
Group 2	.13	.13	.27	.18
Group 3	.08	-.28	-.33	-.54
Group 4	-1.22	-.67	-.92	-.21
Group 5	-1.61	-1.12	-1.77	-1.46
Data Set 1 (with extreme scores)				
Group 1	.46	.79	.81	.91
Group 2	.36	.15	.17	-.07
Group 3	-.06	-.33	-.33	-.28
Group 4	-1.08	-.64	-.88	-.61
Group 5	-1.60	-1.80	-1.54	-1.61

The tests designed to investigate the effects of extreme scores involved data sets 1 and 2, five groups and multiple usage. The effect of introducing two extreme and opposite sets of student scores was nearly always to marginally increase the homogeneity of the final groupings (the exception being for Groupal C with data set 1). The number of omissions also showed little change.

The influence of the extreme scores occurred in the initial stages of the calculation of the seed points by the proportionate division method, which method involved dividing up the range proportionately to the density of the points. The range is the maximum distance between any two students. For example, the inclusion of the extreme points increased the range from 7.061 to 9.766, for data set 1 and Groupal B. This in turn had the effect of generating new seed points which are shown in Table 4-53. It seems that the inclusion of the two opposite and extreme scores in data set 1 had a minimal effect overall in Groupals A and C comparative performances.

Groupals B and D were more susceptible to the inclusion of the extreme scores, because of the change in range, the consequent change in seed points and the resultant formation of different initial groups. The initial group memberships are very influential in determining the skills to be assigned as can be seen in Tables 4-54 to 4-57.

TABLE 4-54

EFFECTS OF EXTREME STUDENT SCORES ON GROUPINGS FORMED BY GROUPAL A

Data Set	Distance After 1st Iteration	Number of Iterations	Distance after 1st Iteration	Number of Omissions	Final Distance	Mean Final Distance	Skills Assigned
Data Set 1	190.465	6	149.406	0 + 0	152.032	1.434	3, 6, 5, 1, 2
Data Set 1 (with extremes)	186.807	12	143.696	0 + 0	149.663	1.411	3, 6, 5, 1, 2
Data Set 2	181.093	9	157.037	5 + 3	161.960	1.652	5, 2, 1, 4, 3
Data Set 2 (with extremes)	179.920	6	159.257	4 + 5	152.130	1.568	5, 2, 1, 4, 3

TABLE 4-55

EFFECTS OF EXTREME STUDENT SCORES ON GROUPINGS FORMED BY GROUPAL B

Data Set	Distance After 1st Iteration	Number of Iteration	Distance after 1st Iteration	Number of Omissions	Final Distance	Mean Final Distance	Skills Assigned
Data Set	151.814	11	139.034	0 + 0	148.280	1.398	3, 4, 3, 6, 1
Data Set 1 (with extremes)	166.832	6	133.839	1 + 1	137.089	1.318	3, 3, 6, 4, 3
Data Set 2	151.449	18	137.940	7 + 3	148.786	1.549	4, 2, 5, 5, 1
Data Set 2 (with extremes)	143.904	6	134.212	9 + 0	152.251	1.569	4, 5, 2, 5, 3

TABLE 4-56

EFFECTS OF EXTREME STUDENT SCORES ON GROUPINGS FORMED BY GROUPAL C

Data Set	Distance After 1st Iteration	Number of Iterations	Distance after 1st Iteration	Number of Omissions	Final Distance	Mean Final Distance	Skills Assigned
Data Set 1	190.609	9	149.890	0 + 0	153.721	1.431	3, 6, 5, 1, 2
Data Set 1 (with extremes)	186.888	6	147.367	0 + 1	152.198	1.449	3, 6, 5, 1, 2
Data Set 2	181.464	10	157.455	5 + 3	159.209	1.624	5, 2, 1, 4, 3
Data Set 2 (with extremes)	180.487	8	158.028	4 + 4	153.593	1.567	5, 2, 1, 4, 3

TABLE 4-57

EFFECTS OF EXTREME STUDENT SCORES ON GROUPINGS FORMED BY GROUPAL D

Data Set	Distance After 1st Iteration	Number of Iterations	Distance after 1st Iterations	Number of Omissions	Final Distance	Mean Final Distance	Skills Assigned
Data Set 1	169.792	9	178.589	2 + 0	179.908	1.729	4, 3, 3, 3, 3
Data Set 1 (with extremes)	153.792	5	161.577	9 + 3	157.033	1.670	3, 3, 3, 4, 4
Data Set 2	165.814	12	156.792	5 + 3	159.974	1.632	1, 2, 6, 3, 5
Data Set 2 (with extremes)	153.981	13	156.055	2 + 0	159.499	1.533	6, 2, 5, 2, 3

Effects of Various Proportions of Eligibility

Tables 4-58 to 4-62 refer to a set of four tests performed on each grouping algorithm in an attempt to identify any effects varying degrees of eligibility may have on the effectiveness of each of the algorithms.

Four sets of data were considered, each with a different average eligibility.

(i) Data Set 0 (100% eligibility, unit 4 student data)
(ii) Data Set 1 (75% eligibility, unit 4 student data)
(iii) Data Set 2 (50% eligibility, unit 4 student data)
(iv) Data Set 5 (33% eligibility, unit 4 student data)

As usual, all tests involved forming five groups with multiple usage of skills permitted.

All algorithms showed a distinct trend to yield more omissions as the average eligibility in the data decreased. The percentages of students omitted by each algorithm in each data set are shown below in Table 4-58.

TABLE 4-58

PERCENTAGE OF STUDENTS OMITTED BY EACH ALGORITHM

Data Set	Groupal A	Groupal B	Groupal C	Groupal D
0, 100% elig.	0%	0%	0%	0%
1 75% elig.	0%	0%	0%	2%
2 50% elig.	7.5%	9%	7.5%	7.5%
5 33% elig.	14%	18%	15%	30%

The same trend of increasing omissions is also evident in the omissions caused by ineligibility for the skills chosen by the algorithms. Omissions caused by the application of size constraints (applied after eligibility constraints in each of the algorithms) showed somewhat the same trend, other than for Groupal B (Table 4-60).

Groupals A, B and C showed a trend of decreasing homogeneity (increasing mean final distance) with decreasing average ineligibilities.

This feature was probably caused by the increasing influence of the eligibility constraints which more severely restricted the relocations possible. It is the relocations of students within the eligibility constraints which progressively increase the homogeneity of the groups. Groupal D behaved irratically in producing more homogeneous groups for data of decreasing average eligibilities (Table 4-62). Again the large number of omissions produced by Groupal D cast doubts on the comparability of these distance measures.

TABLE 4-59

EFFECTS OF DIFFERENT ELIGIBILITY DATA ON GROUPS FORMED BY GROUPAL A

Data Set	Omissions Due to eligibility	Omissions Due to size	Final Distance	Mean Final Distance
Data Set 0 (100% elig)	0	0	149.759	1.412
Data Set 1 (75% elig.)	0	0	152.032	1.434
Data Set 2 (50% elig.)	5	3	161.960	1.652
Data Set 5 (33% elig.)	12	3	152.241	1.672

TABLE 4-60

EFFECTS OF DIFFERENT ELIGIBILITY DATA ON GROUPS FORMED BY GROUPAL B

Data Set	Omissions Due to eligibility	size	Final Distance	Mean Final Distance
Data Set 0 (100% elig.)	0	0	141.735	1.337
Data Set 1 (75% elig.)	0	0	148.280	1.398
Data Set 2 (50% elig.)	7	3	148.786	1.549
Data Set 5 (33% elig.)	18	1	142.658	1.639

TABLE 4-61

EFFECTS OF DIFFERENT ELIGIBILITY DATA ON GROUPS FORMED BY GROUPAL C

Data Set	Omissions Due to eligibility	size	Final Distance	Mean Final Distance
Data Set 0 (100% elig.)	0	0	149.681	1.411
Data Set 1 (75% elig.)	0	0	153.721	1.450
Data Set 2 (50% elig.)	5	3	159.209	1.624
Data Set 5 (33% elig.)	12	4	150.620	1.673

TABLE 4-62

EFFECTS OF DIFFERENT ELIGIBILITY DATA ON GROUPS FORMED BY GROUPAL D

Data Set	Omissions eligibility	Due to size	Final Distance	Mean Final Distance
Data Set 0 (100% elig.)	0	0	139.034	1.311
Data Set 1 (75% elig.)	2	0	179.908	1.729
Data Set 2 (50% elig.)	5	3	159.974	1.632
Data Set 5 (33% elig.)	24	8	109.891	1.485

Comparison of Teacher Generated Groupings With Computer Generated Groupings

In an attempt to answer the second research question: "Are groups formed by the computerized grouping procedure more homogeneous than teacher created groups on selected student characteristics and when both groupings meet the same constraints?"

Three different teacher generated groupings were compared with the corresponding computer generated groupings. The leader of unit 4 selected all grouping parameters applied in these tests; these grouping parameters are listed below.

Test 1

Curricular area: Study Skills, WDRSD

Skills to be considered and eligibilities: C_{11} (31 eligible) D_{10} (8 eligible) D_{14} (19 eligible), E_{14} (5 eligible)

Student Characteristics considered:

Visual Language (mean = 27.25)
Auditory Language (mean = 30.39)
Oral Expressive (mean = 28.73)
Oral Written (mean = 28.07)

Number of students to be grouped: 88

Number of groups to be formed: 2

Size of groups: 1-30, 1-30

Usage of Skills: Single

The results obtained on the above grouping for both the teacher procedure and the computerized procedure are shown in Table 4-63. The table shows the two groupings to have been identical. To test the hypothesis of independence between the two sets of skills groups formed by the two procedures a chi-square test was performed. The chi-square test of independence between these two groupings yielded a χ^2 of 167.689 for 4 degrees of freedom, which is significant beyond the .005 level and as a consequence the hypothesis of independence between the two groupings was rejected. The corresponding phi coefficient (ϕ), a measure of the strength of association, was .976, a very strong association. This unanticipated result occurred because no students were eligible for both skills C_{11} and D_{14}. Consequently no relocations

TABLE 4-63

COMPARISON OF TEACHER GENERATED GROUPS AND COMPUTER GENERATED GROUPS-STUDY SKILLS, WDRSD

Procedure	Omissions	Final Distance	Mean Final Distance	Skills Assigned	Sizes of Groups	Group 1 VL	Group 1 AL	Group 1 ED	Group 1 EW	Group 2 VL	Group 2 AL	Group 2 ED	Group 2 EW
Teachers Procedure	39	94.509	1.928	C11,D14	30, 19	.04,	-.05,	-.12,	.11	-.23,	.03,	.07,	-.25
Computerized Procedure	39	94.509	1.928	C11,D14	30, 19	.04,	-.05,	-.12,	11	-.23,	.03,	.07,	-.25

were possible to maximize the homogeneity of the two groups by Groupal A. This lack of overlap between the two skills with the greatest eligibilities prevented a useful comparison of the two procedures. The nature of the eligibility data and the grouping constraints requested precluded any other possible grouping.

Test 2

Curricular area: Comprehension, WDRSD

Skills considered and eligibilities: CD02(14), CD04(15), CD05(10), CE03(17), CF01(5).

Student characteristics considered: as for test 1

Number of students to be grouped: 88

Number of groups to be formed: 4

Size of groups: 1-25, 1-25, 1-25, 1-25

Usage of Skills: Single

The results obtained from both the teacher procedure and the computerized procedure are shown in Table 4-64.

The eligibilities for the skills selected were very small, having an average eligibility of 14%. Consequently, a large number of omissions resulted (44%) and the groupings produced could not be considered useful. Realistically another set of skills would likely be requested on which to base the grouping. These low eligibilities restricted the number of relocations possible and hence the degree of homogeneity produced by the computerized procedure. The mean final distance of 1.819 however was less than that produced by the teachers' procedure 1.884. This difference reflected a 4% decrease in distance

TABLE 4-64

COMPARISON OF TEACHER GENERATED GROUPS AND COMPUTER GENERATED GROUPS-COMPREHENSION, WDRSD

Procedure	Omissions	Final Distance	Mean Final Distance	Skills Assigned	Sizes of Groups	Group 1 (CE03)	Group 2 (CD04)	Group 3 (CD02)	Group 4 (CD05)
						VL AL ED EW	VL AL ED EW	VL AL ED EW	VL AL ED EW
Teachers Procedure	45	81.019	1.884	CE03, CD04, CD02, CD05	16,15,8, 4	-.01, .25, .04, -.21	-.43, -.21, .44, -.26	.27, -.22, .21, -.10	.27, .22, -.2.1, .52
Computerized Procedure	45	78.247	1.819	CE03, CE04, CD02, CD05	16,10, 10,7	.12, .25, .04, -.21	-.29, -.40, .44, -.56	.18, -.32, .01, .09	-.42, .44, .02, .65

or a 4% increase in homogeneity by the computerized procedure. It should be noted, however, that although teachers had available data on the student characteristics, they did not use it when forming their groups.

The numbers of omissions produced by the two procedures were equal. This result was not unexpected because of the very low eligibilities and small amount of overlap in eligibilities.

Of the 88 students, 79 were placed into the same skill groups and only 9 students were placed in different skill groups, that is into groups assigned different skills. This gave an agreement ratio of 79/88 or .898. This ratio indicated that 79 students out of a possible 88 students were placed into groups assigned the same skill by both procedures. A chi-square test was conducted to test the hypothesis of independence between the two sets of skills groups. The chi-square test of independence yielded a χ^2 of 210.691 for 12 degrees of freedom, which was significant beyond the .005 level and as a consequence the hypothesis of independence between the two groups was rejected. The corresponding phi coefficient (ϕ) was .893, indicating a strong association between the results of the two groupings.

The profiles of the groups produced by the two procedures are shown in Tables 4-65 and 4-66. The two sets of profiles appeared to be similar.

TABLE 4-65

PROFILES OF GROUPS RECOMMENDED BY TEACHERS, TEST 2

Characteristic	Group 1 (CE03)	Group 2 (CD04)	Group 3 (CD02)	Group 4 (CD05)
Visual Language	medium	low	medium/high	medium/high
Auditory Language	medium/high	medium/low	medium/low	medium/high
Expressive Oral	medium	high	medium/high	medium/low
Expressive Written	medium/low	medium/low	medium	high

TABLE 4-66

PROFILES OF GROUPS RECOMMENDED BY THE COMPUTERIZED PROCEDURE, TEST 2

Characteristic	Group 1 (CE03)	Group 2 (CD04)	Group 3 (CD02)	Group 4 (CD05)
Visual Language	medium	medium/low	medium	low
Auditory Language	medium/high	low	medium/low	high
Expressive Oral	medium	high	medium	medium
Expressive Written	medium/low	low	medium	high

Test 3

Curriculum area: DMP

Topics considered and eligibilities: Topic 49 (37), Topic 54 (27), Topic 55 (30) Topic 45 (40)

Student characteristics considered:
Visual numerical (mean = 31.82)
Auditory numerical (mean = 28.68)

Number of students to be grouped: 88

Number of groups to be formed: 5

Size of groups: 15-25, 15-25, 15-25, 15-25, 15-25

Usage of Topics: Single

The results obtained from both the teacher procedure and the computerized procedure are shown in Table 4-67.

The eligibilities for the skills requested were somewhat more useful for grouping purposes than the eligibilities in the two previous tests. The average eligibility was 38% and the number of omissions from the teachers procedure was 12 and 13 from the computerized procedure.

These higher eligibilities and the number of overlaps in students' eligibilities increased the opportunity for the computerized procedure to produce more homogeneous groups than in the earlier two tests. The mean final distance of 0.904 from the computerized procedure compared to a mean final distance of 1.331 from the teachers procedure reflecting a 33% decrease in distance or a 33% increase in homogeneity by the computerized procedure. Again it should be noted that teachers did not use student characteristics in forming their groups.

TABLE 4-67

COMPARISON OF TEACHER GENERATED GROUPS AND COMPUTER GENERATED GROUPS, DMP

Procedure	Omissions	Final Distance	Mean Final Distance	Skills Assigned	Sizes of Groups	Group 1		Group 2		Group 3		Group 4		Group 4	
						VN	AN	VN	AN	VN	AN	VN	AN	VN	AN
Teachers Procedure	12	102.528	1.349	56,49,55, 54,56	17,19,16, 8,17	.17	-.15	.12	.23	.03	-.03	.28	.01	.03	-.01
Computerized Procedure	13	67.793	0.904	56,49,55, 54,56	13,25,14, 13,10	.62	.81	-.17	.37	-.51	.06	.59	-.29	-.15	-1.49

The number of omissions produced by the teachers procedure was 12 as compared to 13 by the computerized procedure. The extra omission was caused by Group 2 (skill 49) exceeding its size limit and selecting a student for relocation who was ineligible for any other skill. This student therefore was placed in the omissions group with the diagnostic "removed from Group 2 because of size constraints."

Of the 88 students, 50 students were placed into the same skill groups and 38 were placed into different skill groups. This gave an agreement ratio of 50/88 or .568 indicating 50 out of a possible 88 students were placed into groups assigned the same skill, considerably less than obtained in Test 2 (comprehension). The corresponding chi-square test of independence between the two groupings yielded a χ^2 of 159.017 for 25 degrees of freedom, which was significant beyond the .005 level. The corresponding phi coefficient (ϕ) was .601, indicating a moderate association between the results of the two groupings. The profiles of the groups produced by the two procedures are shown in Tables 4-68 and 4-69. The two sets of profiles appeared to be somewhat different. It is noticeable that the profiles of the groups produced by teachers show a uniformity of student characteristics. This uniformity was not nearly as noticeable in the groups formed by the computerized procedure.

The computer printout produced as part of Test 3 is shown in Appendix H, page 396. Student names have been omitted for reasons of privacy.

TABLE 4-68

PROFILES OF GROUPS RECOMMENDED BY TEACHERS, TEST 3

Characteristics	Group 1 (Topic 56)	Group 2 (Topic 49)	Group 3 (Topic 55)	Group 4 (Topic 54)	Group 5 (Topic 56)
Visual numerical	medium	medium	medium	medium/ high	medium
Auditory numerical	medium	medium/ high	medium	medium	medium

TABLE 4-69

PROFILES OF GROUPS RECOMMENDED BY THE COMPUTERIZED PROCEDURE, TEST 3

Characteristic	Group 1 (Topic 56)	Group 2 (Topic 49)	Group 3 (Topic 55)	Group 4 (Topic 54)	Group 5 (Topic 56)
Visual numerical	high	medium	low	high	medium
Auditory numerical	high	medium/ high	medium	medium/ low	very low

Teachers' Perceptions of the Computerized Procedure

A questionnaire was prepared and administered to the five teachers of unit 4 in an attempt to ascertain their perceptions of the computerized grouping procedure. The primary purpose of the questionnaire was to provide information useful in answering the third research question: "Do teachers involved in the grouping of students perceive the computerized groupings as being a more efficient procedure than those procedures currently employed and being able to take into account (a) realistic constraints on the formation of groups and (b) relevant learner characteristics?"

The questionnaire, which is shown in appendix G was of two parts. Part I was designed to identify the features of a computerized grouping which teachers considered important. Part 2 was designed to identify the extent to which these same teachers perceived the computerized grouping procedure as including those features which they considered important. Each part comprised 23 parallel questions. For example, Question 10 of Part A asked respondents whether teachers should be able to specify learner characteristics on which to form groups. Question 10 of Part B asked these same respondents to assess how successful the computerized grouping procedure was in allowing teachers to specify relevant learner characteristics. Twelve of the questions referred to options, for example, options for specifying single or multiple usage of skills. Five other questions referred to the use of student characteristics when forming groups. Three questions referred to the omission of students from groups, two questions to the comparative efficiency of the computerized grouping procedure and one question to the format of the computerized reports.

Responses to each question were made on a 5-point rating scale and respondents were asked to make comments where they wished to elaborate on their responses.

In Part A, a score of 1 corresponded to "very desirable" and a score of 5 to "undesirable." In part B, the correspondence was 1 for "very successful" and 5 for "unsuccessful".

The questionnaires were issued to teachers on May 27, 1976 and received back by June 4, 1976 just prior to the end of the school year. Prior to, and while completing the questionnaires, teachers had available the three sets of groupings produced by the computer. What follows is a qualitative description of the responses made by the five teachers who had been introduced to the computerized grouping procedure. These perceptions or impressions reported by teachers are organized under the headings:

(i) options,
(ii) format of grouping reports,
(iii) omissions,
(iv) student characteristics,
(v) efficiency.

Options That teachers should be able to specify the number of groups was consistently considered very desirable or desirable by all respondents, who unanimously agreed that the computerized procedure did this very successfully. The importance of specifying the exact sizes of groups was given as overall median rating by respondents with one respondent claiming it to be an undesirable feature. Respondents also gave a median rating to the computerized procedure's success in providing this option, only two teachers recognizing the procedure's capability for doing so. The option of specifying group sizes as ranges was unanimously considered very important and with one exception as being very successfully achieved by the computer.

Also considered unanimously very important was the need to specify a set of skills from which those to be studied by each group are selected. Teachers did not consider it as important to specify what particular skills were to be assigned to each particular group. Respondents perceived that the computer very successfully achieved this latter goal. Only one respondent noted that the computerized procedure did not provide the option of specification of particular skills to particular groups. The single/multiple usage option was consistently perceived to be desirable and successfully provided.

Teachers generally considered it desirable to have the option of being able to specify particular students who are to be or are not to be placed on the same group. The respondents however gave a median rating to the computerized procedure's success at placing into different groups incompatible students. The latter option of placing specified students in the same group was correctly noted as not being available by four of the respondents.

Teachers did not consider it very important or important that they be able to request groupings based only on student characteristics and without reference to eligibility or to request groups formed only on the basis of eligibilities and without reference to student characteristics. Generally, teachers perceived these options as being successfully provided by the computerized system.

Considered important was the ability to request groups from a subset of the units as well as the whole unit . The option was perceived as being successfully provided by the computerized procedure.

Student Characteristics. With one exception teachers considered the specification of student characteristics on the basis of which groups are to be formed, to be very important but gave only a median rating to the computer's ability to provide this successfully, one respondent giving a rating of 5 (unsuccessful). Another respondent in his accompanying comments stated he preferred to use his personal knowledge of students rather than quantified information on learning styles, which he believed to be inaccurate.

Teachers considered the forming of maximally homogeneous groups as important but gave only a median rating to the computerized procedures ability to provide these groups.

Strangely, teachers did not perceive as important the need to readily observe differences in group profiles and gave a median rating to this feature. A similar rating was given to the computerized procedure's ability to provide this information. Teachers did however consider it important that similarities in learning characteristics over each group should be helpful to them when they prepare instructional prescriptions. The respondents gave a median rating to the procedure's success at providing this help.

Format of Grouping Report

The inclusion in the report of:

(i) names of students in alphabetical order,

(ii) the number of students in each group,

(iii) the name of the skill assigned to the group, and

(iv) the mean of each characteristic for the group, were all considered very important and very successfully achieved by the computerized procedure.

Omissions

Minimizing the number of students omitted from the grouping was generally considered important or very important. However, only one respondent considered the computerized procedure as being successful in achieving this minimization, the overall rating being 3 (the median). Considered very important was the need to provide reasons for each omission and to provide alternative grouping recommendations for these omitted students. The computerized procedure was rated successful in providing reasons for occasions and given a median rating for providing alternative recommendations.

Efficiency

That the computerized grouping procedure should be more efficient (take less staff time) than either a manual grouping procedure using McBee cards or a CMI grouping procedure using Instructional Grouping Recommendation Forms was consistently considered very important by respondents. These respondents considered the computerized procedure to be much more efficient than a manual procedure and as being more efficient than the present CMI system of using Instructional Grouping Recommendation Forms.

From the responses made by the five teachers to questions about the desirability of certain features of a computerized grouping procedure and to questions about the computerized procedure's success in providing these features, it is clear that no feature discussed was considered unimportant. Ten out of the twelve options referred to were considered as very important or important, the least important features being requests for groups of exact sizes and for groupings not based on eligibilities for skills.

Overall, the computerized grouping procedure did not receive top ratings for its perceived success in achieving features considered important. However, no feature possessed by the computerized procedure received less than a median rating over all respondents. The options perceived as being least successfully provided were (i) specifying particular students to be placed in the same group (considered important by most respondents) and (ii) forming groups only on the basis of student characteristics.

Although teachers perceptions of the importance of the various aspects of using student characteristics when forming groups were inconsistent, all aspects referring to maximizing the homogeneity of groups and the use of group profiles in making instructional prescriptions were considered as important. Respondents however consistently gave only median ratings to the computerized procedure's success in providing for these features related to student learning characteristics.

All aspects of the grouping report were considered important and very well met in the computerized reports.

All aspects of minimizing omissions and of providing information helpful in the subsequent placement of these students were also considered very important. The computerized grouping procedure's success in minimizing omissions was rated 3 (the median) by teachers and only slightly above the medianon providing recommendations for those students omitted.

The computerized grouping procedure was perceived as being much more efficient than a manual procedure and more efficient than other WIS-SIM procedures.

Summary

This chapter reported and analyzed data collected as part of the evaluation of the effectiveness of the four algorithms which had been designed and computerized for use in forming groups for instructional purposes. Initially it was determined that Groupal C was most effective with a weight of 0.5 on each skill eligibility and that Groupal D was most effective with a weight 1.0 on each skill eligibility. Groupal A was selected as being the most effective in terms of the number of omitted students and in terms of the homogeneity of the groups formed. This decision was reached on the basis of a testing program of 64 tests in which the parameters of the grouping process were systematically varied to provide a comprehensive range of realistic grouping situations. Over all tests Groupal B most consistently yielded the most homogeneous groups and Groupal A, the least omissions. Considering both criteria of equal importance Groupal A had a higher total ranking and hence was selected for comparison with teacher groupings.

Each of the four algorithms were subjected to a battery of tests to determine the effects of various elements of the grouping process on the groups formed. These parameters included weights of skills, single/multiple usage of skills, size constraints, number of groups, methods of selecting seed points, extreme student scores and different proportions of eligibility. These analyses provided information on the performance of the algorithms which may be useful in their more effective use.

Two of the three comparisons between teacher generated groupings and computer generated groupings failed to produce useful information. The third comparison, which involved the DMP program and related student characteristics, suggested that the computerized procedure yields an equivalent number of omissions and much more homogeneous groups.

Teachers perceptions of the computerized procedure were assessed by a questionnaire in the form of a rating scale. No grouping feature considered important by teachers was considered to be unsuccessfully provided by the computerized procedure. In fact, no feature received less than a median success rating.

CHAPTER V

REVIEW, FINDINGS, RECOMMENDATIONS AND IMPLICATIONS

This chapter summarizes the study as a whole, integrates the findings, makes recommendations based on these findings and finally considers the implications of the findings. The first section provides a summary of Chapters I-IV. The second section synthesizes the findings related to the research questions. The third section contains recommendations on the usage of the computerized procedure. The final section discusses the implications of the findings for research and administrative practice.

Review

This study was concerned with the formation of groups of students and specifically addressed the problem: Can a computerized procedure be developed which is useful in forming groups of students for instructional purposes?

The procedure developed to solve this problem was mathematical in nature and involved utilizing computer technology in its implementation. The study itself comprised the design, development, application and evaluation of the procedure.

The solution to the above problem was based on an analysis of the specific educational environment within which the solution was applied. The procedure developed in this study aimed to facilitate, in part, the management of a particular individualized program of instruction, namely Individually Guided Education (IGE). Specifically, the study was conceived as an extension of the grouping procedures employed by the Wisconsin System for Instructional Management (WIS-SIM), the computer system which supports IGE.

The significance of the problem was derived from a consideration of some features of individualized instructional programs and in particular was supported by (i) the central role of grouping practices in individualized instructional programs, (ii) the need for providing instructional decision-makers with more relevant information on which to base groupings, and (iii) the need to provide more efficient and effective procedures in the formation of the groups.

Chapter I in part considered the educational environment in terms of (i) the purposes of grouping within individualized programs of instruction, (ii) manual grouping practices in IGE, and (iii) factors on which to form groups such as aptitudes, achievement, interests, learning style and the measurement of these factors. This examination led to the setting of the following criteria which an acceptable computerized grouping procedure should meet.

Criterion 1. A computerized grouping procedure should provide for the creation of maximally homogeneous groups based on relevant student learning characteristics.

To provide for flexibility of grouping arrangements compatible with IGE practices the next two criteria were proposed.

Criterion 2: A computerized grouping procedure should permit the storage of diverse data from which selections can be made to meet different instructional purposes.

Criterion 3: A computerized grouping procedure should permit the formation of groups, the sizes and numbers of which can be specified by those responsible for the formation of the groups.

The examination of both IGE and WIS-SIM educational policies and grouping practices made obvious the need to consider the nature of hierarchically sequenced, instructional programs. This consideration led to the following proposal.

Criterion 4: A computerized grouping procedure should take into account the prerequisite structure of the instructional program when such prerequisites help determine the composition of the groups to be formed. Concern for the heterogeneous nature of the variables on which the grouping was to be based led to the next criterion.

Criterion 5: A computerized grouping procedure should permit the selection of data measured on different scales as this is considered relevant to the purpose of grouping.

A brief summary of quantitative models used for grouping in some non-educational areas was presented as part of Chapter I in an attempt to ascertain their relevance to the problem of grouping students for instructional purposes. This summary, which was preliminary to a detailed examination of those techniques considered as being most relevant in the solution of the problem, included:

(i) hierarchical techniques,
(ii) optimization - partitioning techniques,
(iii) density or mode-seeking techniques,
(iv) clumping techniques,
(v) other methods which did not fall clearly into any of the other four groups, for example, factor analysis and discriminant function analysis.

It became apparent from the survey that none of the available clustering techniques were exactly applicable to the grouping situation as defined by Criteria 1 through 5. For example, none of the techniques reviewed directly referred to eligibility for group membership. Neither did any of the applications of these techniques incorporate the option of prespecifying the sizes of the groups. However the partitioning techniques, which are very similar to the steepest descent algorithms used for unconstrained optimization problems in non-linear programming, appeared more directly amenable to such constraints, than did the other clustering techniques, which are mostly used where naturally occurring clusters are sought.

Despite the limitation of providing only local optima, partitioning techniques appeared to most closely meet criterion 1 through 5. Their general structure appeared more adaptable to meet these criteria. It therefore seemed profitable to limit a more detailed examination of clustering techniques to the optimization-partitioning techniques. This was accomplished in Chapter II and served as the basis of the design of an acceptable algorithm.

An examination of the operations research literature revealed that the sub-optimal partitioning techniques introduced in Chapter I were a subset of a wider collection of optimization procedures designed to solve combinatorial problems. Because the problem investigated involved (i) the search for an algorithm directly applicable to the grouping of students for instructional purposes and (ii) the possible subsequent modification of an existing algorithm, it appeared appropriate to review the wider collection of combinatorial procedures and their application to assignment problems. The various procedures reviewed included (i) complete enumeration; (ii) integer programming, (iii) implicit enumeration procedures, and (iv) heuristic or sub-optimal procedures. This review led to the following series of recommendations which constituted the basis of the design of the computerized grouping procedure.

Recommendation 1. Complete enumeration of all groupings to identify that grouping which achieves maximal homogeneity should not be considered further, since it is not a feasible procedure.

Because of the uncertainty of obtaining optimal solutions with integer programming methods and the complex nature of the problem being studied (e.g., eligibility and size constraints on group membership) the following recommendation was also made.

Recommendation 2: An integer programming procedure should not be considered further as a viable method of identifying that grouping which achieves maximal homogeneity.

Although the problem of grouping students for instructional purposes may be formulated in terms of the implicit enumeration procedures such as branch and bound, backtracking and dynamic programming their application to this grouping problem appeared to be computationally infeasible; this observation led to the following recommendation:

Recommendation 3: None of the exact procedures should be considered further as viable methods for identifying that grouping which achieves maximal homogeneity.

Because it appeared that the combinational problem being investigated could only be reasonably solved by a sub-optimal approximation the following recommendation was made.

Recommendation 4: The available heuristic algorithms should be reviewed for the purposes of selecting one algorithm for implementation, or alternatively selecting desirable characteristics of different algorithms to comprise a new algorithm.

Most of these techniques employ three distinct procedures:

(i) procedures for initiating groups,
(ii) procedures for relocating entities, and
(iii) a grouping criterion

Because the purpose of grouping students was to produce groups of individuals maximally homogeneous in relation to the total set of variables and also to ensure that each individual was relatively similar to every other individual in the same cluster on each variable, because the grouping was less oriented towards the objectives of classification or clustering than towards the purposes of dissection, and because of the administrative restrictions in the formation of groups, the following recommendation was made:

Recommendation 5: A minimum variance criterion should be used as part of a heuristic-programming technique.

Twelve minimum variance procedures were reviewed and a decision made to consider further the partitioning procedures of Forgy, Jancey, MacQueen and Ball and Hall as well as variants on them proposed by Wishant and McRae (page 112). This discussion resulted in the following recommendations.

Recommendation 6: Seed points leading to an initial partition should scan the whole data set and take into account the density of the data set.

Recommendation 7: The Forgy reallocation procedure should be used to produce a local minimum of the total within groups sum of squares criterion. The algorithm utilized nearest centroid sorting with fixed numbers of groups.

Recommendation 8: All variables on which the grouping is to be based should be standardized.

Recommendation 9: The measure of similarity to be used is the weighted Euclidean metric

$$d_{ij} = \left(\sum_{k=1}^{h} a_k (x_{ik} - x_{jk})^2 \right)^{\frac{1}{2}}$$

where a_k is the weight attached to the kth variable.

Recommendations 4 through 9 referred to homogenizing procedures usually considered in the literature independently of any administrative constraints.

Accordingly, the homogenizing procedure outlined had not been considered within the operating framework imposed by the constraints. Without further development, such an algorithm was considered inadequate for solving the problem of grouping students. Because of the structure searching purposes of clustering algorithms, the individualistic nature of heuristic algorithms, and the lack of applications of computerized grouping procedures in school settings, it was concluded that no presently existing algorithm could be directly employed to solve the grouping problem investigated. It, however, was the case that some features of these other procedures could serve as the basis of a design for an algorithm useful in grouping students for instructional purposes.

The development of an acceptable algorithm concerned the fitting of a homogenizing procedure within a framework of administrative constraints. It was considered that such an algorithm should comprise the following essential elements (it was assumed the number of groups to be formed was known):

(i) a criterion to be optimized,

(ii) a measure of inter-student similarity,

(iii) the determination of seed points around which to form groups,

(iv) the allocation of students to groups on the basis of learning characteristics,

(v) the continued reallocation of students to groups to optimize the criterion,

(vi) the allocation of skills to groups,

(vii) the allocation of size limits to groups,

(viii) the imposition of group size constraints,

(ix) the imposition of eligibility constraints.

Four Algorithms

On the basis of the foregoing considerations, which were developed in Chapters I and II, four algorithms were designed, and computer programs were written, implemented and evaluated.

The first grouping algorithm (Groupal A):

(i) initially selected skills,

(ii) matched group sizes with skills,

(iii) allocated eligible students to these groups to maximize their homogeneity, and then

(iv) applied other constraints.

The second grouping algorithm (Groupal B):

(i) initially allocated students to groups to maximize homogeneity and without any constraints,

(ii) then on the basis of these groups selected skills,

(iii) and finally applied other constraints.

The third grouping algorithm (Groupal C) was Groupal A modified to include student eligibilities (weighted) with student characteristics in the assignment of these students to groups.

The fourth grouping algorithm (Groupal D) was Groupal B modified to include student eligibilities (weighted) with student characteristics in the initial allocation of students to groups.

In each of the four procedures the users specified:

(i) whether the eligibility for skills was to be taken into account,

(ii) whether student characteristics were to be taken into account,

(iii) whether the one skill could be studied by more than one group,

(iv) the number of groups to be formed,

(v) the size of each group to be formed,

(vi) the number of students to be grouped,

(vii) the skills to be considered,

(viii) the student characteristics to be considered,

(ix) the students to be placed in different groups,

(x) the maximum number of iterations permitted in the relocation process.

For Groupals B and C the user also specified:

(xi) the weighting to be applied to the skills,

(xii) whether seed points were to be calculated or specified.

The profile of each recommended instructional group comprised

(i) the group number,

(ii) the skill to be taught,

(iii) the number of students in each group,

(iv) the group members identification numbers and names in alphabetical order,

(v) the distance of each student from the mean, and

(vi) the mean, variance and standard deviation of each student characteristic for each group.

An omissions group was also provided showing those students omitted from the grouping and the reason for each omission.

The Evaluation Procedures

The evaluation was performed in three parts. The first part determined the most effective of the four procedures developed; the second part determined which of the computerized grouping procedure or a teacher grouping procedure was the most effective, and the third part determined teachers' perceptions of the efficiency and effectiveness of the computerized grouping procedure.

The complete evaluation plan designed to answer the three research questions involved (i) establishing sets of criteria, (ii) designing a testing program in which the different grouping procedures are tested under different conditions, and (iii) collecting sets of student data on which to test the different procedures.

The data required for these testing purposes consisted of data relating to the eligibility of students for particular skills of an objective based program and data relating to the characteristics of each student. One unit of 106 students from an IGE school was chosen as the source of student data and the teachers in this unit provided an assessment of the computerized procedure.

Data collected on the learner characteristics of each student included measures on nine constructs of learning style, the number of skills mastered in the Study Skills component of the WDRSD program and scores on the Stanford Diagnostic Test of Reading. These data served as the basis for the compilation of four different sets of student characteristics which were then combined with three simulated sets of eligibility data to form seven different data sets for use in the comparison of the four grouping procedures.

The testing program designed to evaluate the effectiveness of the four algorithms utilized a subset of four student characteristics and six skills. The testing program was prepared in three parts:

(i) 14 tests to determine the most effective weights for Groupals C and D,

(ii) 64 tests to determine the most effective of the four algorithms,

(iii) 12 tests to determine the most effective method of determining seed points in Groupals B and D.

Throughout the complete testing program of 90 tests, the data sets, group sizes, number of groups and the single and multiple usage option were all varied to represent the conditions in which the grouping procedures would realistically be implemented. The testing program however, involved the selection of a relatively small set of conditions from a much larger set of possible conditions.

Two criteria were used as the basis of selection among the weights and among the algorithms:

(i) the average final distance as a measure of homogeneity, and

(ii) the number of students omitted from groups.

Both criteria were considered of equal importance and all tests were accorded an equal value. The performances of each weight and each algorithm were ranked on each test and each criteria and the weight and the algorithm with the greatest ranked scores over all tests and both criteria were selected as being the most effective.

The comparison between the teacher generated groupings and the computer generated groupings involved three separate groupings, each for a different instructional program. The unit leader selected all parameters for each grouping: skills and student characteristics to be considered, numbers of groups, sizes of groups, and single or multiple usage of skills. The number of omissions yielded by each method was recorded. The homogeneity of each set of groups was measured by the average final distance. The similarity of each set of groups was assessed by comparing the profiles of the groups formed by a ratio of agreement, and by the Chi-Square statistic and the phi coefficient of association.

The teachers' perceptions of the efficiency and effectiveness of the computerized grouping procedure were obtained by having the five teachers involved complete a questionnaire. The questionnaire was designed in two parts; part 1 was designed to identify those features of a computerized grouping procedure considered to be important by teachers, and part 2 was designed to identify the extent to which teachers perceived the computerized grouping procedure as having been successful in providing these features.

Chapter 4 presented the analyses of these evaluations.

Findings

This section is presented in six parts. First, findings related to the effectiveness of different weights, applied to the skills eligibilities in Groupals C and D are described. Second, the findings related to the effectiveness of the four algorithms are described. Thirdly, the effects of various elements of the grouping procedures on the groups formed are discussed. This section is followed by a fourth which concerns the effects of weighted skill eligibilities, of single/multiple usage of skills, of size constraints, of the numbers of groups, of different methods of selecting seed points, of extremes in student data, and of various proportions of eligibility. The fifth section describes the findings related to the comparison between the teacher generated and computer generated groupings. The final section reports the findings related to the teachers evaluation of the computerized grouping procedure.

Selection of Weights

Fourteen tests were designed to determine which of seven weights ranging from 0.5 to 20.0 were the most effective when applied to skills eligibilities in Groupals C and D. The same seven weights were each applied to data set (75% eligibility) and data set 2 (50% eligibility) with number of students to be grouped, number and sizes of groups requested and multiple usage of skills all being held constant.

The analysis showed that a weight of 0.5 applied in Groupal C was clearly the most effective in terms of consistently yielding the least number of omissions and the most homogeneous groups. Generally, it was found that the homogeneity of the groups varied indirectly with the size of the weight. However, the rate of decrease in homogeneity became less as the weight increased. The largest decreases in homogeneity occurred for weights of 2.0 or less. The number of omissions varied only slightly over both data sets, with a weight of 0.5 yielding the least. Consequently it was decided to utilize a weighting of 0.5 for all skills eligibilities in later usages of Groupal C.

The corresponding tests of weights applied in Groupal D did not reveal any one weight as being the most effective although a weight of 1.0 was chosen for later implementation because of its very slight advantage on both the omissions and homogeneity criteria. This lack of a single most effective weight resulted from two trends each of which nullified the other's effectiveness. An increase in weight tended to correspond to a decrease in homogeneity and a decrease in the number of omissions. The decrease in homogeneity (measured only on student characteristics and not including the skill eligibilities) was apparently a function of the stronger influence of the increasing weights in the location-relocation process. Conversely, the true student characteristics contributed a decreasing influence in the location-relocation process; the end locations consequently possessed less homogeneity than they did with lighter weights. The trend to lesser numbers of omitted students for increased weights was anticipated, this being a purpose in the design of Groupal D. The influence of the weighted skill eligibilities was felt in the initial assignment of students to groups when students with similar patterns of eligibilities were forced into the same initial groups.

A weight of 1.0 was chosen to be applied to skill eligibilities in later tests of Groupal D on the basis of the evidence which indicated its comparatively effective performances on both criteria.

Selection of the Most Effective Algorithm

A set of 64 tests was designed to determine that algorithm which most consistently produced the least number of omissions and the most homogeneous groups. The tests included five different data sets, three different sets of group sizes and three different numbers of groups to be formed. The number of students to be grouped was the only constant element throughout the testing program.

Over all tests, Groupals A and B most consistently gave the most homogeneous groupings with Groupal B being slightly more effective. Groupal A was clearly the most effective algorithm in most consistently yielding the least number of omissions. Considering both criteria of equal importance, Groupal A was selected for a later comparison with a teacher grouping procedure.

Several exceptions to the trends relating to Groupal A should be noted. Groupal A was comparatively ineffective in yielding the most omissions when no group size constraints were applied. In this case the small lower limit of the group sizes (1-99) resulted in the multiple usage of the one skill and the selection of only two skills over all five groups. This reduced selection of skills resulted in more students being omitted because of ineligibilities for the two skills.

The number of omissions yielded by Groupals A and B were generally unaffected by the presence of extreme student scores but Groupals C and D clearly gave less omissions in the presence of extreme scores and in comparison to Groupals A and B. This advantage of Groupal D, however was not consistent, and reflected the volatile behavior of the algorithm in being affected by the extreme scores, which in turn affected the seed points and initial group membership, and finally the assignment of skills. Groupal D, throughout the testing program produced inconsistent results, a feature not shared by Groupal A.

Groupal A was comparatively more effective in providing homogeneous groups on data with high average eligibility than on data with low average eligibility. All algorithms produced more homogeneous groups in data containing higher average eligibility, and Groupal A's comparative disadvantage on low eligibility data is difficult to interpret because of the much greater numbers of omissions (particularly produced by Groupals B and D) which have the effect of grouping quite different subsets of students out of those in the original unit.

The selection of the weights for skill eligibilities and the final selection of the most effective algorithm were based on only a relatively small sample of conditions. Although the testing program was considered to be representative of realistic grouping situations, it should be recognized that the final selections were based on one testing program and one set of criteria. The reliability of these selections is unknown.

An analysis of the effects of varying different elements in the grouping while controlling other elements was also provided in Chapter IV. The purpose of this analysis was to identify any unforeseen effects which would lead to recommendations for using the procedures and perhaps to later modifications.

Effects of Weights

In an attempt to more specifically describe the effects of weights on the groups formed by Groupals C and D eight tests were performed involving data set 2, multiple usage of skills and five groups of sizes 25-30, 25-30, 25-30, 15-20, 5-10.

There was no observable relationship between the numbers of students omitted and the sizes of the weights applied in Groupal C. However, the homogeneity of the final groups was always the least when no weight was applied to the skills eligibilities. The light weighting of 0.5 resulted in only slightly less homogeneous groupings. The decrease in homogeneity for larger weights was noted earlier. Another effect of these heavier weights was to reduce the number of iterations required to achieve convergence to a local optimum level of homogeneity. This more efficient result was due to the influence of the weighted eligibilities in initially placing into the same groups students with similar patterns of eligibilities and consequently restricting the number of relocations. This lesser number of relocations led to a quicker convergence to a local optimum, with, unfortunately, also a lower level of homogeneity.

The convergence was also noted as not being a continuous decrease in homogeneity, when this was measured on the true student characteristics. This was due to the inclusion of weighted eligibilities in the location-relocation process and their inclusion in the measurement of homogeneity.

The most noticeable changes in final group profiles produced by Groupal C resulted by using the lower weights of 0.0, 0.5, and 1.0 with only very minor changes being noticed as a result of heavier weights. The smaller groups were the most suceptible to changes in profile as a result of the changes in weights.

The weights applied to Groupal D possessed some trends similar to those for Groupal C in terms of irregular patterns of convergence, decreasing numbers of iterations for increasing weights, and decreasing homogeneity for increasing weights. The trends however, were not as strong as for Groupal C, the inconsistency due to the initial assignment of skills and group sizes which were based on subsets of overall eligibilities. Groupal D results displayed a weak trend of decreasing eligibilities for increasing weights.

The profiles of the final groups produced by Groupals C and D were dissimilar, making clear that these algorithms produced quite different sets of groups.

Single and Multiple Usage of Skills

Three different data sets were used to compare the effects of single usage of skills and multiple usage of skills. Although each involved different proportions of eligibility, separate skills eligibilities in each set were fairly uniform. As a consequence of these uniformities the difference between the greatest eligibility and lower limit of the corresponding group size always failed to exceed the next greatest eligibility. Therefore in the tests on Groupals A and C, no skill was selected more than once. Consequently, no information was obtained on the differential effects of single and multiple usage of skills for Groupals A and C.

Because Groupals B and D assigned skills on the basis of greatest eligibility within each group, the same skill was found to be assigned more than once in every application of the multiple usage option. In all comparisons of single and multiple usages with Groupal B, single usage produced fewer omissions but multiple usage produced slightly more homogeneous groups. Groupal D also tended to assign skills more than once, to yield fewer omissions for single usage, and to produce slightly more homogeneous groups for multiple usage. This trend however was weak.

Group Sizes

Typically the number of groups and the sizes of groups are inversely related for a fixed number of students, and so in these cases it is difficult to attribute changes in final groupings to the effects of group size rather than to the number of groups. However, in this set of tests, the number was held constant and the sizes of the groups altered from 25-30, 25-30, 25-30, 15-20, 5-10, to 30, 25, 20, 16, 15 to five groups each of size 1-99. The three sets of sizes therefore involved range, exact and unconstrained specifications.

The critical influence of the lower limit of the group size was again experienced when the effects of different group sizes on the final groupings were examined. In the unconstrained case (1-99) and with Groupals A and C, the skill with the greatest eligibility was assigned to groups until the difference between the lower limit of the first group (1) and the highest eligibility became less than the second greatest eligibility. Conversely, in the case of exact sizes, the increase in the lower limit (from 25-30 to 30-30) tends to prevent the multiple assignment of skills because the difference between the lower limit and the greatest eligibility tends not to exceed the second greatest eligibility, as was the case with the data set 1.

As a consequence of the multiple assignment of skills in the unconstrained case the number of omissions increased, these students being ineligible for the much reduced set of skills assigned. As was to be expected, the removal of size constraints resulted in more homogeneous groups, although these gains were slight. This was also the case when group sizes were expressed exactly.

Similar trends in homogeneity were observed for Groupals B and D. However, the specification of exact sizes resulted in more omissions because of size constraints particularly for Groupal B. This was a consequence of the decrease in the upper limit of two of the larger groups (e.g. from 25-30 to 25-25) and the multiple assignment of skills to groups.

Numbers of Groups

All four algorithms showed a trend to yield fewer omissions for increasing numbers of groups. Groupals A and C consistently yielded less omissions than either Groupals B or D.

Algorithms A, B and C all showed a trend to produce more homogeneous groups for increasing numbers of groups. Groupal D behaved erratically, perhaps because of the influence of the large number of omissions.

Methods of Selecting Seed Points

Groupals B and D both possessed the option of user specification of seed points. The effects of the algorithmic selections of seed points were compared with the effects of (i) random selection, (ii) systematic selection, and (iii) teacher selection of seed points. Systematic selection involved the selection of the first five students in alphabetical order as seed points.

The analysis showed that for data set 1, multiple usage and five groups the proportionate division method of Groupal B yielded less omissions (zero) than any other method. The proportionate division method also produced the most homogeneous groups, although this advantage over any other method was small. It was also observed that the distances after the final iteration ranged from 133.675 for a random selection to 139.034 for Groupal B's selection, a difference of

47%. These results suggest that the local optimum produced by Groupal B may be a good approximation of the global optimum.

The inclusion of skills eligibilities as student characteristics in Groupal D distorted the trends reported above for Groupal B because the proportionate division produced the least homogeneous groupings and the modal number of omissions. This unpredictable behavior of Groupal D, characteristic of its performances throughout the evaluation, is a direct consequence of the inclusion of skills eligibilities as student characteristics and their exclusion in the measurement of homogeneity.

Extreme Student Data

The inclusion of two opposite and extreme vectors of student scores had a minimal effect on the omissions and homogeneity of groups produced by Groupals A and C. Groupals B and D were more susceptible to the inclusion of the extreme scores because of the change in range between the two most distant students, the consequent change in seed points and the resultant formation of different initial seed points. The initial group memberships are very influential in determining the skills assigned to groups.

Varying Proportions of Eligibility

In a series of 16 tests run on four sets of data each with a different proportion of eligibilities it was noted that all algorithms showed a distinct trend to yield more omissions as the average eligibility decreased. Groupal A produced the least omissions for all data sets.

Groupals A, B and C each showed a distinct trend towards decreasing homogeneity for decreasing average eligibility. Groupal D again performed erratically.

Some Tentative Trends

Although the testing program designed for the evaluation of the four algorithms was intended to be comprehensive and representative of realistic grouping situations, it nevertheless was a small sample of such situations. This observation particularly applied to the series of tests designed to determine the effects of different elements of the process. Accordingly, any findings can only be tentative and may very well be data dependent. Despite these limitations several strong trends in the results produced by the algorithms were noted. The following list of trends is based on the results of the testing program.

(i) Groupals B and D were the most unstable of the algorithms, primarily because of the assignment of skills on the basis of subsets of eligibilities. Groupal D, which included weighted skills eligibilities as student characteristics was the most unstable of all algorithms because of this influence of weighted eligibilities.

(ii) For Groupal C (really Groupal A with weighted skills eligibilities included as student characteristics) the number of iterations and the degree of homogeneity varied indirectly as the size of the weights. The rate of decrease in homogeneity declined rapidly for weights greater than 2.0.

(iii) Convergence occurred in all tests for all algorithms, distance decreasing continuously in Groupals A and B but not in Groupals C and D.

(iv) The multiple assignment of a skill to groups in Groupals A and C was dependent upon the distribution of eligibilities and the lower limits of the group sizes. Groupal B was better designed to yield multiple assignment of the same skills.

(v) The removal of size constraints resulted in slight improvements in homogeneity for all algorithms.

(vi) For all algorithms, the number of omissions was indirectly related to the number of groups.

(vii) For Groupals A, B and C, increasing the number of groups increased the homogeneity of the groups.

(viii) For Groupal B, the proportionate division method of selecting seed points was the most effective of the methods compared in yielding the least omissions and the most homogeneous groups.

(ix) The inclusion of extreme scores had only minor effects on the groupings produced by Groupals A and C, but the groupings produced by Groupals B and D were more susceptible to extreme student characteristics.

(x) For all algorithms, the number of omissions was indirectly related to the average eligibility.

(xi) For Groupals A, B and C, homogeneity decreased with decreasing average eligibility.

Comparison of Teacher Generated Groupings With Computer Generated Groupings

Three different groupings were made on which to compare the effectiveness of a teacher grouping procedure and Groupal A.

The first grouping, based on four skills of the Study Skills component of the WDRSD program, produced inconclusive evidence in as much as identical sets of groups were produced by the two procedures. This peculiar result is explained by the complete lack of overlap in student eligibilities for the two skills selected by both procedures. The nature of the teacher specified grouping constraints precluded any other possible grouping.

The second test involved forming groups based on five skills of the Comprehension component of the WDRSD program. The computerized grouping procedure produced slightly more homogeneous groups than did the teacher grouping procedure. Both procedures produced an equal number of omissions. The small improvement in the homogeneity of the groups produced by the computerized procedure (a 4% improvement) can be explained by the very low average eligibility (14%) of the data. This low proportion of eligibility restricted the number of relocations possible and hence the degree of homogeneity produced by the computerized procedure.

Consequently, the groups produced by both procedures were very similar; that this was so is indicated by a high agreement ratio of .898 (the agreement ratio compared the number of students placed into the same skills groups to the total number of students to be grouped). A phi coefficient of .893 was also obtained, this indicating a strong association. As well as these statistical comparisons the profiles of the groups in each set appeared similar.

The third test provided conditions more conducive to an examination of the comparative effectiveness of the two grouping procedures. The test was based on four topics of the DMP program and possessed an average eligibility of 38%. The teacher procedure produced one less omission than did the computerized procedure, the extra omission being caused by one group exceeding its size constraint and the consequent selection of a student for relocation who was ineligible for any other skill.

The computerized procedure produced much more homogeneous groups than did the teacher procedure. The considerable improvement of 33% was due to the greater proportion of eligibility and the subsequent greater number of relocations possible. Consequently the two groupings showed less similarity, as was indicated by an agreement ratio of .568 and a phi coefficient of .601 indicating only a moderate level of association. These indications of a moderate association are supported by an inspection of the group profiles produced by each of the two procedures. The profiles of the teacher generated groups were noticeably uniform whereas the profiles of the computer generated groups were noticeably varied, indicating the possibility of basing instructional prescriptions on the strengths of the student characteristics possessed by these groups.

Although it is difficult to make generalizations based on the comparisons reported in this section, it appears likely that the computerized grouping procedure will produce more homogeneous groups than a manual procedure. This trend is likely to be more noticeable where the proportion of eligibilities permits the formation of useful groups.

Teacher Perceptions of the Computerized Procedure

Following the tests on the comparative effectiveness of the teacher and computerized grouping procedures, the five teachers of unit 4 responded to a questionnaire (Appendix G) designed to determine whether these teachers considered the computerized procedure as being more efficient, in terms of time spent on the grouping process, than their manual procedure. Teachers' perceptions of the success of the computerized procedure in taking into account realistic contraints and relevant learner characteristics were also sought. Respondents rated the importance of various features of the computerized grouping procedure and also the success which they perceived the procedure had in providing these features.

None of the features and options provided by Groupal A were considered unimportant by teachers. Ten out of the twelve options were considered very important or important, the least important being requests for groups of exact sizes and for groupings not based on eligibilities for skills. Overall, the computerized grouping did not receive top ratings for its perceived successes in achieving features considered important. However, no feature possessed by the computerized procedure received less than a median success rating over all respondents.

All aspects of using student characteristics to maximize the homogeneity of groups and the use of group profiles in making instructional prescriptions were considered as important. Respondents however consistently gave only median ratings to the computerized procedure's success in providing homogeneous groups based on relevant student characteristics. All aspects of minimizing omissions and providing information helpful in the subsequent placement of these students were also considered very important. Again, the computerized procedure's perceived success in minimizing omissions and providing alternative recommendations received only median ratings.

The computerized grouping procedure was perceived to be much more efficient than a manual procedure and more efficient than current WIS-SIM procedures.

The teachers' perceptions of the computerized procedure were based on a 45 minute introductory explanation of the procedure and an examination of the print-outs in the comparison of the teacher and computerized procedures. The inadequacy of this preparation was borne out by the inaccurate perceptions of some respondents in attributing features to the computerized procedure which it did not possess and conversely, in not recognizing features it did possess. Such observations tend to lessen the confidence which can be placed in the information collected on teacher perceptions.

Recommendations

The recommendations which follow refer to future applications of the computerized grouping procedure developed as part of this study. Recommendations are made in the areas of:

(i) modifications to existing features,
(ii) inclusion of new features and options, and
(iii) evaluation.

The significance of the research undertaken in this study was partially supported by the need to provide more efficient and more effective procedures in the formation of groups for instructional purposes. It is this need and the desire to make the procedure more flexible which motivates the following recommendations. The recommendations refer primarily to Groupal A, the algorithm selected as being the most efficient.

Modifications to Existing Features

Efficient and effective applications of Groupal A require a selection of skills which contains sufficient eligibilities to make possible the fulfillment of the user's requests. Therefore, a summary of eligibilities for those skills requested should be available. This information is available in the Skills-Eligibility Profile, one of the WIS-SIM reports. Alternatively, this initial information may be made available as a preliminary print option in Groupal A. On the basis of this initial information, a decision can be made by the user (or the program) whether to continue with the grouping.

Alternatively, the main print option, the set of group profiles, may be extended to contain records of students' characteristics. These considerations lead to the following recommendation.

Recommendation 1. Separate print options should be available for

(i) a summary of skills eligibilities,

(ii) listing of student records, either in raw score or standardized form,

(iii) group profiles expanded to include individual student's characteristics. The groups referred to include the omissions group.

These should be the only print options provided.

The assignment of skills to groups directly effects the number of omissions. The selection of skills on the basis of greatest eligibility does not take into account the pattern of eligibilities across students. That is, it does not consider overlapping eligibilities where the one student is eligible for more than one skill. Only one basic process of assigning skills was used in this study and consequently little can be said of its comparative effectiveness. An alternative method is to select skills in the order of greatest single eligibilities, these being the number of students eligible for a particular skill and no others. Size constraints could be applied as a 1-1 correspondence between sizes and single eligibilities, both arranged in descending order. This is but one alternative which nevertheless prompts the next recommendation.

Recommendation 2. Various heuristic methods for assigning skills to groups should be compared within the framework of Groupal A to determine a more effective method of minimizing omissions. One method should be the greatest single eligibility method.

The influence of the lower size limits of groups even when sizes were unconstrained, was seen as critical in the multiple assignment of skills to groups. To be assigned again, the difference between the eligibility of the skill already assigned and the lower limit of the group had to exceed the next greatest eligibility. This is the only function of each lower size limit. Results obtained from this procedure are, of course, data dependent, but the procedure itself seems rational and worthy of further implementation. Nevertheless, the unsatisfactory results in the unconstrained case require an alternative to the skill selection process for the multiple usage option. This alternative could involve single eligibilities (as in Recommendation 2), skills being reassigned if the difference between the greatest two single eligibilities is greater than the lesser of these, and so on. This is but one alternative which leads to the next recommendation.

Recommendation 3. Various methods for assigning skills as part of the multiple usage option should be compared within the framework of Groupal A. One method may be the greatest single eligibility method. Because of the possible unfamiliarity of users with standardized scores it may be useful to report mean student characteristics scores in raw form. Consequently the following recommendation is made.

Recommendation 4. The option should be provided for the reporting of student characteristic scores in either original score form or standardized score form.

No recommendations are made concerning the proportional division method for determining seed points, the local optimization method, or the rule for relocating students when size constraints are enforced, because it is considered that these basic elements of the computerized procedure are appropriate.

New Features and Options

Although only one selection of seed points by teachers resulted in less homogeneous groupings than did the proportional division method in Groupal B, the provision of the option of permitting user specification of seed points enhances the flexibility of the procedure, allows the user more direction in the grouping and may be of use where some students can be identified as being representatives of groups of similar students. Consequently the following recommendation is made.

Recommendation 5. The option of user specification of seed points should be provided.

Although the weighting of skill eligibilities did not improve the effectiveness of Groupals A and B, it did suggest the possible utility of weighting student characteristics. The selection of the characteristics to be weighted and the size of the weight would be at the discretion of the user. Although weighting of variables is somewhat controversial in the clustering literature its provision in the grouping of students like the specification of seed points may provide for more user direction in the grouping process and more flexibility in using the procedure.

Accordingly, the following recommendation is made.

Recommendation 6. The option should be provided which permits the weighting of student characteristics.

The following two recommendations are made as a result of being considered by teachers to be important features of a computerized grouping procedure.

Recommendation 7. An option should be provided which permits selected students to be placed in the same group.

Recommendation 8. An option should be provided whereby selected students can be excluded from the grouping.

Recommendations 5-8 do not affect the basic structure of Groupal A, but rather increase its potential usefulness and flexibility. The remaining recommendation does not refer to the design of the procedure, but to its evaluation.

Evaluation

Neither the reliability nor the validity of the findings reported in this chapter have been clearly established. Although the testing program attempted to be comprehensive, the tests of the effectiveness of procedures and the tests of the effects of different elements in the grouping process lack in generalizability and the results are conditional upon the testing situation. Consequently, a more comprehensive evaluation plan is required to test the effectiveness and utility of the computerized grouping procedure. Therefore the following recommendation is made:

Recommendation 9.

Desirable features of a subsequent evaluation include:

(i) Testing of the effects of different elements of the grouping process should involve simulated sets of data for eligibilities and student characteristics. These data should include low eligibilities, around 20% average eligibility.

(ii) More extensive computer procedure and teacher procedure comparisons should be made over a wide range of grouping conditions.

(iii) The student data should not only relate to learning styles but should include other data thought to be useful by teachers.

(iv) User perceptions of the effectiveness and utility of the computerized grouping procedure should be sought after users have had the opportunity to make extensive comparisons between the two procedures.

Implications

The methodology developed and implemented as part of this study has immediate application only in those schools served by a computerized instructional management system. Such a system is WIS-SIM, which has been designed to service the instructional management needs of IGE schools. The implications of this study therefore may be considered in terms of

(i) implications for the administration of instructional programs, and

(ii) implications for research.

The aim of the computerized grouping procedure was to facilitate in part the management of individualized programs of instruction. Some evidence has been presented that the procedure is likely to be both more effective and more efficient than manual or semi-automated procedures. As such it can make the attainment of IGE goals more feasible and specifically may allow the teacher more readily to adapt instruction to differences among students. The system permits the rapid processing of large quantities of data unmanageable by manual means. That the procedure provides this service efficiently has been suggested by the results of this study. However, the quality of the information provided to educational decision makers remains largely unknown. Many questions arise as a consequence of these doubts, which may be read as needs for further investigations of the outcomes produced by the computerized procedure. Are the end goals of individualized education better met by the adoption of an automated procedure for forming instructional groups? That is, are the achievements of the students involved improved as a consequence of being members of maximally homogeneous groups, when these are based on relevant learner characteristics. This basic question has not been addressed in this study but should be considered because of its fundamental importance.

If indeed a computerized grouping procedure is more efficient in terms of less teacher time spent on grouping students, then this saving in time can be expected to result in some organizational and operational changes in the school implementing the system. Does the frequency of forming groups change by adopting the computerized system? Do staff spend less time on clerical tasks associated with grouping students and more time on planning and instructional tasks? Does its adoption cause any role changes? How is the instructional decision-making process affected by the use of the computerized grouping system? Undoubtedly the adoption of a computerized grouping procedure will have implications in these administrative areas. Hopefully, gains in the efficiency and effectiveness of the instructional process will be made; but this will only be made clear by a careful monitoring and evaluation of subsequent administrative effects.

The utilization of the methodology developed in this study permits the use of large amounts of quantified information. A real possibility exists, however, that the measurement of constructs such as the various dimensions of learning style may be considered as reliable and valid,when in fact those used in this study have not yet been shown to possess these desirable characteristics. The computerized grouping procedure is very much dependent upon the quality of the student data for its effectiveness in helping attain the goals of the instructional programs. The need for quantified data on student learning characteristics will perhaps be reinforced by the availability of an automated grouping procedure.

Consequently, users must be aware that the real constraints on the effectiveness of the scheme are imposed by the relevancy of the data and the accuracy of its measurement. Serious implications for using such a procedure concern the ability of the instructional decision-maker to utilize relevant and appropriate information on which to form groups. The selection of factors on which to form groups should be based on considerations of parisomony, relevance and discrimination, all of which require truly professional judgments. This information may not be readily available in usable form. The adoption of a computerized grouping procedure seems to imply more data collection, storage and processing.

Sound judgments about the instructional process need to be complemented by a full appreciation of the various options possessed by the computerized procedure (assuming the inclusion of the recommended features of the previous section). The effectiveness of the grouping is dependent upon a knowledgeable selection of options, for example, what weightings of characteristics to use, what seed points to use, as well as other more obvious choices as to what skills to request groupings on, multiple or single usage of skills and what students to place in the same group. Generally, it appears that the computerized grouping procedure is more sophisticated than other manual procedures and consequently will require a period of adjustment and familiarization by potential users. The development of useful documentation and inservice procedures is a subsequent step. An important principle to be emphasized to users is that groupings generated by the computer are recommended groupings and are subject to acceptance or modification by teachers, the final instructional decision-makers.

A computerized grouping procedure is perhaps best provided as part of a generalized instructional management system. The procedure developed in this study was implemented independently of WIS-SIM, but may be considered as an extension of WIS-SIM's present grouping system, which does not attempt to form groups in accordance with user specified constraints. If the computerized grouping procedure is to be implemented as part of WIS-SIM, some integration of the two grouping systems would be necessary with the deletion of some present WIS-SIM forms e.g. the Instructional Grouping Recommendation Group Report, the Instructional Grouping Recommendation (Summary) Report and the Instructional Grouping Recommendation (Omissions) Report (Appendix F). Alternatively, if these reports are retained only one additional report showing the groups formed would be required. This report should contain information similar to that of the group profiles developed in this study.

Although the purpose of the computerized grouping procedure was to form student groups for instructional purposes, especially in the environment of individualized instructional programs, the options available as part of the procedure make its application to other instructional environments and other non-instructional purposes possible.

The appropriate choice of options may permit its use in programs which do not possess a pre-requisite structure. The identification of groups of students with similar exceptional qualities may be possible for diagnostic purposes or counseling purposes. The use of the grouping procedure on a school wide basis, or district wide basis is also feasible in the grouping of students for instructional or non-instructional purposes, for example, in the assignment of students to schools on the basis of shortest distance and taking into account various administrative constraints. The extended uses of the computerized grouping procedure to non-instructional areas may therefore be a fruitful exercise.

The application of the grouping procedure developed as part of this study is but one application of a statistical and computerized procedure in the solution of an educational problem, that of efficiently and effectively grouping students for instructional purposes. This first step should now be followed by a pilot test of its efficiency and effectiveness and subsequently by a more extensive evaluation of its instructional and administrative effects.

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APPENDICES

APPENDIX A

COMPUTER PROGRAM FOR GROUPAL A

```
C---- GROUPING ALGORITHM A ----

      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=15
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     *        NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     *        SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION XNAME(4,MAXSTU),VLEARN(MAXLV,MAXSTU)
      DIMENSION ID(MAXSTU)
      DIMENSION XMEAN(MAXLV),XSTDEV(MAXLV),XVAR(MAXLV)
      DIMENSION NUM(MAXSK)
      DIMENSION NSKNOS(MAXSK),NLVNOS(MAXLV)
      INTEGER   SKNAME(3,MAXSK),LVNAME(3,MAXLV)
      DIMENSION NINGRP(MAXGRP)
      DIMENSION NICNOS(10)
      LOGICAL INCOM


C ------------------------------------------------------------------
C     INPUT AND ECHO PARAMETER CARDS
      WRITE(6,1)
    1 FORMAT('1','GROUPING ALGORITHM A',/,1X,20('-'),/)

      READ(5,2)NG,NSKILL,NLER,NTOT,ILIMIT
    2 FORMAT(I2,1X,I2,1X,I2,1X,I3,6X,I2)
      WRITE(6,3)NG,NSKILL,NLER,NTOT
    3 FORMAT('0NUMBER OF GROUPS REQUESTED = ',I2,/,
     1       ' NUMBER OF SKILLS CONSIDERED = ',I2,/,
     2       ' NUMBER OF LEARNER VARIABLES CONSIDERED = ',I2,/,
     3       ' NUMBER OF STUDENTS = ',I3,/)
      IF (ILIMIT.EQ.0) ILIMIT=10

      READ(5,4)K,L,M
    4 FORMAT(3I1)
      IF(K.EQ.1)WRITE(6,5)
    5 FORMAT(' ELIGIBILITY FOR SKILLS TAKEN INTO ACCOUNT')
      IF(K.EQ.0)WRITE(6,6)
    6 FORMAT(' ELIGIBILITY FOR SKILLS NOT TAKEN INTO ACCOUNT')
      IF(L.EQ.1)WRITE(6,7)
    7 FORMAT(' LEARNER VARIABLES TAKEN INTO ACCOUNT')
      IF(L.EQ.0)WRITE(6,8)
    8 FORMAT(' LEARNER VARIABLES NOT TAKEN INTO ACCOUNT')
      IF(M.EQ.1)WRITE(6,9)
    9 FORMAT(' MORE THAN ONE GROUP MAY STUDY THE SAME SKILL')
      IF(M.EQ.0)WRITE(6,11)
   11 FORMAT(' ONLY ONE GROUP MAY STUDY A PARTICULAR SKILL')

      READ(5,10)(NLOW(I),NHIGH(I),I=1,NG)
   10 FORMAT (20I2)
      WRITE (6,19) (NLOW(I),NHIGH(I),I=1,NG)
   19 FORMAT('0GROUP RANGES = ',10(2I3,2X))
```

```
C ---------------------------------------------------------------------
C     RANGE TEST
      MSUM=0
      NSUM=0
      KLOW=99
      DO 20 I=1,NG
      NSUM=NSUM+NLOW(I)
      MSUM=MSUM+NHIGH(I)
      IF (NLOW(I).GT.NHIGH(I)) WRITE (6,16) I,NLOW(I),NHIGH(I)
   16 FORMAT (' INVALID RANGE  GRP #',I2,' LOW = ',I2,', HIGH = ',I2)
      IF (NLOW(I).GT.KLOW) WRITE (6,18) I,NLOW(I),KLOW
   18 FORMAT (' SEQUENCE ERROR  GPR #',I2,' CURRENT LOW = ',
     1 I2,', PREVIOUS LOW = ',I2)
      KLOW = NLOW(I)
   20 CONTINUE
      IF(NTOT.GT.MSUM.OR.NTOT.LT.NSUM)WRITE(6,30)NTOT
   30 FORMAT(' RANGE SIZES DO NOT CORRESPOND TO TOTAL NUMBER OF STUDEN
     1',I3)


C ---------------------------------------------------------------------
C     READ IN SKILL AND LEARNER VARIABLE NAMES
      READ (5,10) (NSKNOS(I),I=1,NSKILL)
      WRITE (6,185) (NSKNOS(I),I=1,NSKILL)
  185 FORMAT ('0SKILLS CONSIDERED = ',20I3)
      READ (5,10) (NLVNOS(I),I=1,NLER)
      WRITE (6,187) (NLVNOS(I),I=1,NLER)
  187 FORMAT ('0LEARNER VARIABLES CONSIDERED = ',20I3)
      DO 170 I=1,NSKILL
      READ(5,180)(SKNAME(J,I),J=1,3)
  180 FORMAT(3A6)
  170 CONTINUE
      DO 175 I=1,NLER
      READ(5,180)(LVNAME(J,I),J=1,3)
  175 CONTINUE


      WRITE(6,14)
   14 FORMAT ('0',49X,'STUDENT RECORDS')
      WRITE(6,17)
   17 FORMAT('0', 8X,'STUDENT NAME',9X,' VL  VN  AC  AN  KT  SI  SO',
     1' EO  EW   SD   SM  SEX',' 1 2 3 4 5 6 7 8 91011121314'/)

C ---------------------------------------------------------------------
C     READ IN STUDENT RECORDS
      DO 15 I=1,NTOT
      READ(5,12)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
   12 FORMAT(I4,1X,4A6,2X,9F2.0,F3.1,1X,F2.0,F1.0,1X,14I1)
      WRITE(6,13)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
   13 FORMAT(I5,1X,4A6,9F4.0,F8.1,1X,F4.0,2X,F2.0,14I2)
   15 CONTINUE


C -----------------------------------------------------------------
C       LEFT-JUSTIFY SELECTED SKILLS AND LEARNER VARIABLES
        DO 250 I=1,NTOT
        DO 220 II=1,NLER
        KK=NLVNOS(II)
        IF (L.EQ.0) VLEARN(KK,I)=0.0
  220   VLEARN(II,I)=VLEARN(KK,I)
        DO 240 II=1,NSKILL
        KK=NSKNOS(II)
        IF (K.EQ.0) ISKILL(KK,I)=1
  240   ISKILL(II,I)=ISKILL(KK,I)
  250   CONTINUE


C -----------------------------------------------------------------
C       CALCULATE NUMBER OF STUDENTS ELIGIBLE FOR EACH SKILL
        WRITE (6,130)
  130   FORMAT ('0NUMBER OF STUDENTS ELIGIBLE FOR THE FOLLOWING SKILLS -
        DO 140 J=1,NSKILL
        NUM(J)=0
        DO 150 I=1,NTOT
        NUM(J)=NUM(J)+ISKILL(J,I)
  150   CONTINUE
        WRITE(6,160)(SKNAME(II,J),II=1,3),NUM(J)
  160   FORMAT (1X,3A6,' -- ',I3)
  140   CONTINUE


C -----------------------------------------------------------------
C       CALCULATE MEAN, VARIANCE, STD. DEV. OF EACH LEARNER VARIABLE
        WRITE (6,70)
   70   FORMAT ('0 *** LEARNER VARIABLES - MEAN, VAR. AND STD. DEV. ***'
        DO 90 I=1,NLER
        SUMSQ=0.0
        SUM=0.0
        XN=0.0
        DO 80 J=1,NTOT
        IF(VLEARN(I,J).LT.0)GO TO 80
        SUM=SUM+VLEARN(I,J)
        SUMSQ=VLEARN(I,J)**2+SUMSQ
        XN=XN+1.
   80   CONTINUE
        IF (XN .LT. 0.5) XN=1.0
        XMEAN(I)=SUM/XN
        XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
        XSTDEV(I)=SQRT(XVAR(I))
   90   WRITE(6,100)(LVNAME(J,I),J=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  100   FORMAT(' ',3A6,' HAS MEAN = ',F6.2,2X,'VAR. = ',F8.2,2X,'SD. = '
     1,F6.2)


C ----------------------------------------------------------------------
C     MISSING DATA
      DO 200 I=1,NTOT
      DO 210 J=1,NLER
      IF(VLEARN(J,I).GE.0)GO TO 210
      VLEARN(J,I)=XMEAN(J)
  210 CONTINUE
  200 CONTINUE


C ----------------------------------------------------------------------
C     STANDARDIZE STUDENT VARIABLE SCORES
      DO 95 J=1,NTOT
      DO 97 I=1,NLER
      IF (XSTDEV(I).LT. 0.0001) GOTO 98
      SLEARN(I,J)=(VLEARN(I,J)-XMEAN(I))/XSTDEV(I)
      GOTO 97
   98 SLEARN(I,J)=XMEAN(I)
   97 CONTINUE
   95 CONTINUE

      WRITE (6,214)
  214 FORMAT ('0 *** STANDARDIZED LEARNER VARIABLES ***')
      DO 215 I=1,NTOT
  215 WRITE (6,216)  ID(I),(XNAME(J,I),J=1,4),(SLEARN(J,I),J=1,NLER)
  216 FORMAT (I5,1X,4A6,14(F6.2))


C ----------------------------------------------------------------------
C      ALLOCATE SKILLS TO GROUP BASED ON LARGEST NO. ELIGIBLE
      DO 350 I=1,NG

      MAX=0
      DO 310 II=1,NSKILL
      IF (NUM(II).LE.MAX) GOTO 310
      NSK=II
      MAX=NUM(II)
  310 CONTINUE

      IF (MAX.GE.NLOW(I)) GOTO 330
      WRITE (6,320) I,NLOW(I),MAX
  320 FORMAT('0LOWER LIMIT OF GROUP =',I2,' = ',I3,
     *       ' IS GREATER THAN # OF STUDENTS ELIGIBLE = ',I3)
      NLOW(I)=MAX
  330 NUM(NSK)=MAX-NLOW(I)
      NGSKIL(I)=NSK
      IF (M.EQ.0) NUM(NSK)=0
  350 CONTINUE
      WRITE (6,351) (NGSKIL(I),I=1,NG)
  381 FORMAT ('0SKILLS ASSIGNED TO GROUPS - ',20I3)

C ------------------------------------------------------------------
C      CALCULATE SEED POINT FOR EACH GROUP
      DO 400 I=1,NSKILL

      KOUNT=0
      DO 360 II=1,NG
      IF (NGSKIL(II).NE.I) GOTO 360
      KOUNT=KOUNT+1
      NGNO=II
  360 CONTINUE

      IF (KOUNT.EQ.0) GOTO 400
      IF (KOUNT.EQ.1) GOTO 370
      CALL SEEDY(I)
      GOTO 400

  370 DO 390 J=1,NLER
      XN=0.
      SUM=0.
      DO 380 II=1,NTOT
      IF (ISKILL(I,II).EQ.0) GOTO 380
      XN=XN+1.
      SUM=SUM+SLEARN(J,II)
  380 CONTINUE
  390 SEED(J,NGNO)=SUM/XN
  400 CONTINUE


C ------------------------------------------------------------------
C      OUTPUT GROUP MEANS
      PDIST=0.0
      KKK=0
  450 CONTINUE
      WRITE (6,410)
  410 FORMAT ('0 *** MEAN OF EACH GROUP ***')
      DO 420 I=1,NG
  420 WRITE (6,430) I,(SEED(J,I),J=1,NLER)
  430 FORMAT(' GROUP #',I2,13(F7.3))


C ------------------------------------------------------------------
C      ASSIGN STUDENTS TO SEED POINTS
      TDIST=0.0
      TSSW=0.0
      WRITE (6,470)
  470  FORMAT ('0 *** STUDENT GROUP ASSIGNMENTS ***')
      DO 500 I=1,NTOT
      NSTUNO(I)=0
      DO 490 J=1,NG
      NSK=NGSKIL(J)
      IF (ISKILL(NSK,I).EQ.0) GOTO 490
      X=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (NSTUNO(I).EQ.0) GOTO 480
      IF (X.GT.XDIST) GOTO 490
  480 XDIST=X
      NSTUNO(I)=J
  490 CONTINUE
      IF (NSTUNO(I).EQ.0) XDIST=0.0
      SDIST(I)=XDIST
      TDIST=TDIST+XDIST
      TSSW=TSSW+XDIST**2
  500 CONTINUE
      WRITE (6,495) (I,NSTUNO(I),SDIST(I),I=1,NTOT)
  495 FORMAT (3(7X,'STU #',I3,' ASG TO ',I2,' W. DIST ',F5.2))


C ---------------------------------------------------------------------
C      COMPUTE MEAN AND VAR. OF ALL GROUPS
       DO 600 J=1,NG
       DO 590 I=1,NLER
       SUM=0.0
       XN=C.0
       DO 580 II=1,NTOT
       IF (NSTUNO(II).NE.J) GOTO 580
       SUM=SUM+SLEARN(I,II)
       XN=XN+1.
   580 CONTINUE
       IF (XN .LT. 0.5) XN=1.0
       SEED(I,J)=SUM/XN
   590 CONTINUE
   600 CONTINUE

       IF (KKK.EQ.0) V=TDIST
       IF (KKK.NF.0) V=ABS(TDIST-PDIST)
       KKK=KKK+1
       PDIST=TDIST
       XMSSW=TSSW/FLOAT(NTOT-NG)
       WRITE (6,620: KKK,TDIST,V,TSSW,XMSSW
   620 FORMAT ('0FOR ITERATION #',I2,' TOTAL DISTANCE = ',F7.3,
      1' WHICH DIFFERS FROM PREVIOUS BY ',F7.3,
      2 /,5X,'TOTAL SUM OF SQUARES WITHIN = ',F8.2,
      3   ',  MEAN SUM OF SQUAFES WITHIN = ',F7.2)
        IF (V.GT.0.001 .AND. KKK.LE.ILIMIT) GOTO 450


C ---------------------------------------------------------------------
C       OUTPUT OMISSIONS AND NO. IN EACH GROUP
        DO 630 J=1,NG
    630 NINGRP(J)=0
        DO 640 I=1,NTOT
        JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 640
      NINGRP(JJ)=NINGRP(JJ)+1
  640 CONTINUE
      WRITE (6,655)
  655 FORMAT('0GRP #   SKILL   NO. IN GROUP')
      WRITE (6,660) (J,NGSKIL(J),NINGRP(J),J=1,NG)
  660 FORMAT (I4,4X,I4,8X,I4)

C ----------------------------------------------------------------------
C     CHECK FOR GROUPS OVERLOADED
      ASSIGN 670 TO IRETN
      INCOM=.FALSE.
      NOTYPE=0
  670 CONTINUE
      DO 675 J=1,NG
      IF (NINGRP(J).GT.NHIGH(J)) GOTO 677
  675 CONTINUE
      GOTO 800


C ----------------------------------------------------------------------
C     COMPUTE DISTANCES AND FIND FARTHEST STUDENT IN OVERLOADED GROUP
  677 CONTINUE
      X=0.0
      IX=0
      DO 685 J=1,NG
      IF (NINGRP(J).LE.NHIGH(J)) GOTO 685
      DO 680 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 680
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 680
      X=SDIST(I)
      IX=I
  680 CONTINUE
  685 CONTINUE


C ----------------------------------------------------------------------
C     FIND ANOTHER GROUP FOR THIS STUDENT
  690 CONTINUE
      Y=0.0
      IY=0
      J=NSTUNO(IX)
      DO 700 JJ=1,NG
      IF (J.EQ.JJ) GOTO 700
      IF (NINGRP(JJ).GE.NHIGH(JJ)) GOTO 700
      NSK=NGSKIL(JJ)
      IF (ISKILL(NSK,IX).EQ.0) GOTO 700

C ---------- CHECK FOR INCOMPATIBLES IN SAME GROUP
      IF (.NOT.INCOM) GOTO 690
      DO 695 II=1,NIC
      I=NICNOS(II)
      IF (I.EQ.IX) GOTO 695
      IF (NSTUNO(I).EQ.JJ) GOTO 700
  695 CONTINUE
  698 CONTINUE
      YY=DIST (SLEARN(1,IX),SEED(1,JJ),NLER)
      IF (IY.NE.0 .AND. YY.GT.Y) GOTO 700
      IY=JJ
      Y=YY
  700 CONTINUE
      IF (IY.NE.0) GOTO 750


C ---------------------------------------------------------------------
C     CANNOT MOVE - SO PUT IN OMISSIONS
      WRITE (6,720) IX,J
  720 FORMAT ('0 STUDENT #',I3,' HAS BEEN BOOTED OUT OF GRP #',I2)
      WRITE (6,710) ID(IX),(XNAME(II,IX),II=1,4),J
  710 FORMAT (I5,1X,4A6,' HAS BEEN REMOVED FROM GRP #',I2)
      NSTUNO(IX)=-(NOTYPE+J)
      SDIST(IX)=0.0
      NINGRP(J)=NINGRP(J)-1
      GOTO 760


C ---------------------------------------------------------------------
C     MOVE STUDENT AND RECOMPUTE GROUP MEANS
  750 CONTINUE
      WRITE (6,755) IX,J,IY
  755 FORMAT ('0 STUDENT #',I3,' IS TO BE MOVED FROM GRP #',
     1  I2,' TO GRP #',I2)
      NSTUNO(IX)=IY
      NINGRP(J)=NINGRP(J)-1
      NINGRP(IY)=NINGRP(IY)+1

      DO 790 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 780 I=1,NTOT
      IF (NSTUNO(I).NE.IY) GOTO 780
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  780 CONTINUE
      SEED(JJ,IY)=SUM/XN
  790 CONTINUE

  760 CONTINUE
      DO 775 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 770 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 770
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  770 CONTINUE
      SEED(JJ,J)=SUM/XN
  775 CONTINUE


C --------------------------------------------------------------------
C     RECOMPUTE STUDENT DISTANCES
      TDIST=0.0
      TSSW=0.0
      DO 740 I=1,NTOT
      SDIST(I)=0.0
      J=NSTUNO(I)
      IF (J.LE.0) GOTO 740
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      X=SDIST(I)
      TDIST=TDIST+X
      TSSW=TSSW+X**2
      SDIST(I)=X
  740 CONTINUE
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,745) TDIST,TSSW,XMSSW
  745 FORMAT(5X,'TOTAL DIST. = ',F7.3,' TSSW = ',F8.2,' MSSW = ',F7.2)
      GOTO IRETN


C --------------------------------------------------------------------
C     PROCESS INCOMPATIBLE STUDENTS IN SIMILAR MANNER
  800 CONTINUE
      INCOM=.TRUE.
      NOTYPE=20
      READ (5,810,END=900) NIC,NICNOS
  810 FORMAT (I2,10I4)
      WRITE (6,815) (NICNOS(I),I=1,NIC)
  815 FORMAT ('0INCOMPATIBLE STUDENTS = ',10I5)
      IF (NIC.LE.1) GOTO 800
      DO 830 I=1,NIC
      DO 820 J=1,NTOT
      IF (ID(J).EQ.NICNOS(I)) GOTO825
  820 CONTINUE
      WRITE (6,822) NICNOS(I)
  822 FORMAT (' NO STUDENT FOR ID #',I4)
      GOTO 800
  825 NICNOS(I)=J
  830 CONTINUE

  860 CONTINUE
      DO 840 J=1,NG
      KOUNT=0
      DO 835 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 835
      KOUNT=KOUNT+1
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
  835 CONTINUE
      IF (KOUNT.LE.1) GOTO 840

      X=0.0
      IX=0
      DO 850 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 850
      IF (SDIST(I).LT.X) GOTO 850
      X=SDIST(I)
      IX=I
  850 CONTINUE
      ASSIGN 860 TO IRETN
      GOTO 690
  840 CONTINUE
      GOTO 800


C -----------------------------------------------------------------
C     OUTPUT FINAL TOTALS AND LEAVE
  900 CONTINUE
      DO 950 J=1,NG
      NSK=NGSKIL(J)
      WRITE (6,910) J,NSK,(SKNAME(JJ,NSK),JJ=1,3),NINGRP(J)
  910 FORMAT ('1','GROUP #',I2,', SKILL #',I2,2X,3A6,/,
     1 ' NUMBER OF STUDENTS RECOMMENDED = ',I3,//,
     2 ' STUD #',12X,'STUDENT NAME',10X,'DISTANCE',/)
      DO 940 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 930 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 930
      SUM=SUM+SLEARN(I,II)
      SUMSQ=SLEARN(I,II)**2+SUMSQ
      XN=XN+1.0
      IF (I.NE.1) GOTO 930
      SDIST(II)=DIST(SLEARN(1,II),SEED(1,J),NLER)
      WRITE (6,920) ID(II),(XNAME(JJ,II),JJ=1,4),SDIST(II)
  920 FORMAT (2X,I4,4X,4A6, 5X,F7.3)
  930 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
      IF (I.EQ.1) WRITE (6,70)
      WRITE (6,100) (LVNAME(JJ,I),JJ=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  940 CONTINUE
  950 CONTINUE
```

```
C ---------------------------------------------------------------------
C     OUTPUT OMISSIONS GROUP AND INDICATE WHY
      JJ=0
      WRITE (6,960)
  960 FORMAT ('1','OMISSIONS GROUP',//,' STUD #',12X,
     1 'STUDENT NAMF',15X,'REASON',/)
      DO 990 I=1,NTOT
      J=NSTUNO(I)
      IF (J.GT.0) GOTO 990
      JJ=JJ+1
      IF (J.EQ.0) GOTO 980
      J=-J
      IF (J.LT.NOTYPE) GOTO 970
      J=J-NOTYPE
      WRITE (6,965) ID(I),(XNAME(II,I),II=1,4),J
  965 FORMAT (I5,1X,4A6,5X,'REMOVED FROM GRP #',I2,
     1 ' DUE TO INCOMPATIBILITY')
      GOTO 990
  970 WRITE (6,975) ID(I),(XNAME(II,I),II=1,4),J
  975 FORMAT (I5,1X,4A6,5X,'REMOVED FROM GRP #',I2,
     1 ' DUE TO SIZE CONSTRAINTS')
      GOTO 990
  980 CONTINUE
      DO 100C IS=1,NSKILL
      IF (ISKILL(IS,I).NE.0) GOTO 1010
 1000 CONTINUE
      WRITE (6,985) ID(I),(XNAME(II,I),II=1,4)
  985 FORMAT (I5,1X,4A6,5X,'NOT ELIGIBLE FOR ANY SKILL')
      GOTO 990
 1010 NS=0
      DO 1020 IS=1,NSKILL
      IF (ISKILL(IS,I).EQ.0) GOTO 1020
      NS=NS+1
      NUM(NS)=SKNAME(2,IS)
 1020 CONTINUE
      WRITE (6,1025) ID(I),(XNAME(II,I),II=1,4),(NUM(J),J=1,NS)
 1025 FORMAT (I5,1X,4A6,5X,'NOT ELIGIALE FOR ANY SKILL',
     1 ' SELECTED BUT',/35X,'ELIGIBLE FOR SKILLS ',15A4)
  990 CONTINUE
      WRITE (6,995) JJ
  995 FORMAT ('0NUMBER OF STUDENTS = ',I3,/,'1')
      CALL EXIT
```

```
C---- SEED POINT ROUTINE - ALGORITHM A ----

      SUBROUTINE SEFDY (NSK)
      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=15
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     +       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     +       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION NPART (MAXGRP)


C -------------------------------------------------------------------
C     FIND 2-STUDENTS W. GREATEST PAIR WISE DISTANCE
      NSTU1=1
      NSTU2=1
      DMAX=0.0
      DO 10 I=1,NTOT
      IF (ISKILL(NSK,I).E3.0) GOTO 10
      NN=I+1
      DO 15 II=NN,NTOT
      IF (ISKILL (NSK,II).EQ.0) GOTO 15
      X=DIST (SLEARN(1,I),SLEARN(1,II),NLER)
      IF (X.LE.DMAX) GOTO 15
      NSTU1=I
      NSTU2=II
      DMAX=X
15    CONTINUE
10    CONTINUE
      WRITE (6,17) NSK,NSTU1,NSTU2,DMAX
   17 FORMAT('0SKILL =',I2,', 1ST STUDENT = ',I3,', 2ND STUDENT = ',
     1 I3,', DIFFERENCE = ',F9.3)


C -------------------------------------------------------------------
C     CONSTRUCT A TABLE OF STUDENT DISTANCES
      NELIG=0
      DO 100 I=1,NTOT
      IF (ISKILL(NSK,I).EQ.0) GOTO 100
      X=DIST (SLEARN (1,I), SLEARN(1,NSTU1),NLER)
      IF (NELIG.EQ.0) GOTO 30
      DO 20 II=1,NELIG
      IF (X.LT.SDIST(II)) GOTO 40
20    CONTINUE
30    SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=I
      GOTO 90
40    NSTU2=I
      DO 50 III=II,NELIG
      TEMP=SDIST(III)
      SDIST (III)=X
      X=TEMP
      NTEMP=NSTUNO(III)
      NSTUNO(III)=NSTU2
50    NSTU2=NTEMP
      SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=NSTU2
90    NELIG=NELIG+1
100   CONTINUE
      WRITE (6,80) (SDIST(I),NSTUNO(I),I=1,NELIG)
   80 FORMAT (5(2X,F9.3,2X,I3))


C -----------------------------------------------------------------
C     SUM THE LOWER GROUP LIMITS
      SUM=0.0
      DO 110 I=1,NG
      IF (NGSKIL(I).NE.NSK) GOTO 110
      SUM=SUM+FLOAT(NLOW(I))
110   CONTINUE


C -----------------------------------------------------------------
C     ESTABLISH A PARTITIONING
      NEQSK=0
      DO 120 I=1,NG
      IF (NGSKIL(I).NE.NSK) GOTO 120
      NEQSK=NEQSK+1
      PORTN=(FLOAT(NLOW(I))/SUM)*FLOAT(NELIG)
      IPORTN=PORTN
      IF((PORTN-IPORTN).GE.0.5) IPORTN=IPORTN+1
      NPART(NEQSK)=IPORTN
120   CONTINUE


C -----------------------------------------------------------------
C     BALANCE THE PARTITIONS
      WRITE (6,135) NSK
  135 FORMAT ('0 *** PARTITIONING FOR SKILL #',I2,' ***')
      WRITE (6,145) NELIG,(NPART(J),J=1,NEQSK)
145   FORMAT (' COMPUTED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      DO 140 J=1,NEQSK
      NSUM=0
      DO 130 I=1,NEQSK
130   NSUM=NSUM+NPART(I)
      IF(NSUM.EQ.NELIG) GOTO 150
      IF(NSUM.GT.NELIG) NPART(J)=NPART(J)-1
      IF(NSUM.LT.NELIG) NPART(J)=NPART(J)+1
140   CONTINUE
150   CONTINUE
      WRITE (6,155) NELIG,(NPART(J),J=1,NEQSK)
155   FORMAT (' BALANCED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      IF(NSUM.NE.NELIG) STOP
```

```
C ---------------------------------------------------------------------
C      SELECT MID-POINTS
       I=(NELIG+1)/2
       II=(NELIG+2)/2
       X1=SDIST(I)-SDIST(1)
       X2=SDIST(NELIG)-SDIST(II)


C ---------------------------------------------------------------------
C      FIND MEAN OF EACH PARTITION
       NEOSK=0
       II=0
       DO 200 I=1,NG
       IF (NGSKIL(I).NE.NSK) GOTO 200
       NEOSK=NEOSK+1
       NN=NPART(NEQSK)+II
       N=II+1
       N2=NN
       N1=N
       IF(X1.LT.X2) GOTO 160
       N2=NELIG+1-N
       N1=NELIG+1-NN
160    DO 190 K=1,NLER
       XN=0.
       SUM=0.
       DO 180 J=N1,N2
       JJ=NSTUNO(J)
       SUM=SUM+SLEARN(K,JJ)
180    XN=XN+1.
190    SEED(K,I)=SUM/XN
       II=NN
200    CONTINUE
       RETURN
       END
```

APPENDIX B

COMPUTER PROGRAM FOR GROUPAL B

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```
C---- GROUPING ALGORITHM 8 ----

      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=15
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     *        NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     *        SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION XNAME(4,MAXSTU),VLEARN(MAXLV,MAXSTU)
      DIMENSION ID(MAXSTU)
      DIMENSION XMEAN(MAXLV),XSTDEV(MAXLV),XVAR(MAXLV)
      DIMENSION NUM(MAXSK)
      DIMENSION NSKNOS(MAXSK),NLVNOS(MAXLV)
      INTEGER   SKNAME(3,MAXSK),LVNAME(3,MAXLV)
      DIMENSION NINGRP(MAXGRP)
      DIMENSION NICNOS(10)
      DIMENSION NGELIG (MAXSK,MAXGRP)
      LOGICAL INCOM,SIZEC


C --------------------------------------------------------------------
C       INPUT AND ECHO PARAMETER CARDS
      WRITE(6,1)
    1 FORMAT('1','GROUPING ALGORITHM 8',/,1X,20('-'),/)

      READ(5,2)NG,NSKILL,NLER,NTOT,ILIMIT
    2 FORMAT(I2,1X,I2,1X,I2,1X,I3,6X,I2)
      WRITE(6,3)NG,NSKILL,NLER,NTOT
    3 FORMAT('0NUMBER OF GROUPS REQUESTED = ',I2,/,
     1        ' NUMBER OF SKILLS CONSIDERED = ',I2,/,
     2        ' NUMBER OF LEARNER VARIABLES CONSIDERED = ',I2,/,
     3        ' NUMBER OF STUDENTS = ',I3,/)

      READ(5,4)K,L,M,N
    4 FORMAT(4I1)
      IF(K.EQ.1)WRITE(6,5)
    5 FORMAT(' ELIGIBILITY FOR SKILLS TAKEN INTO ACCOUNT')
      IF(K.EQ.0)WRITE(6,6)
    6 FORMAT(' ELIGIBILITY FOR SKILLS NOT TAKEN INTO ACCOUNT')
      IF(L.EQ.1)WRITE(6,7)
    7 FORMAT(' LEARNER VARIABLES TAKEN INTO ACCOUNT')
      IF(L.EQ.0)WRITE(6,8)
    8 FORMAT(' LEARNER VARIABLES NOT TAKEN INTO ACCOUNT')
      IF(M.EQ.1)WRITE(6,9)
    9 FORMAT(' MORE THAN ONE GROUP MAY STUDY THE SAME SKILL')
      IF(M.EQ.0)WRITE(6,11)
   11 FORMAT(' ONLY ONE GROUP MAY STUDY A PARTICULAR SKILL')
      IF(N.EQ.1)WRITE(6,21)
   21 FORMAT(' SEED POINTS FOR GROUPS COMPUTED')
      IF(N.EQ.0)WRITE(6,22)
   22 FORMAT(' SEED POINTS FOR GROUPS SPECIFIED')

      READ(5,10)(NLOW(I),NHIGH(I),I=1,NG)
   10 FORMAT (20I2)
      WRITE (6,19) (NLOW(I),NHIGH(I),I=1,NG)
   19 FORMAT('0GROUP RANGES = ',10(2I3,2X))


C ------------------------------------------------------------------
C       RANGE TEST
      MSUM=0
      NSUM=0
      KLOW=99
      DO 20 I=1,NG
      NSUM=NSUM+NLOW(I)
      MSUM=MSUM+NHIGH(I)
      IF (NLOW(I).GT.NHIGH(I)) WRITE (6,16) I,NLOW(I),NHIGH(I)
   16 FORMAT (' INVALID RANGE  GRP #',I2,' LOW = ',I2,', HIGH = ',I2)
      IF (NLOW(I).GT.KLOW) WRITE (6,18) I,NLOW(I),KLOW
   18 FORMAT (' SEQUENCE ERROR  GRP #',I2,' CURRENT LOW = ',
     1 I2,', PREVIOUS LOW = ',I2)
      KLOW = NLOW(I)
   20 CONTINUE
      IF(NTOT.GT.MSUM.OR.NTOT.LT.NSUM)WRITE(6,30)NTOT
   30 FORMAT(' RANGE SIZES DO NOT CORRESPOND TO TOTAL NUMBER OF STUDEN
     1',I3)


C ------------------------------------------------------------------
C       READ IN SKILL AND LEARNER VARIABLE NAMES
      READ (5,10) (NSKNOS(I),I=1,NSKILL)
      WRITE (6,185) (NSKNOS(I),I=1,NSKILL)
  185 FORMAT ('0SKILLS CONSIDERED = ',20I3)
      READ (5,10) (NLVNOS(I),I=1,NLER)
      WRITE (6,187) (NLVNOS(I),I=1,NLER)
  187 FORMAT ('0LEARNER VARIABLES CONSIDERED = ',20I3)
      DO 170 I=1,NSKILL
      READ(5,180)(SKNAME(J,I),J=1,3)
  180 FORMAT(3A6)
  170 CONTINUE
      DO 175 I=1,NLER
      READ(5,180)(LVNAME(J,I),J=1,3)
  175 CONTINUE


C ------------------------------------------------------------------
C       READ IN SEED POINTS
      IF (N.EQ.1) GOTO 190
      DO 195 I=1,NG
      READ (5,197) (SEED(J,I),J=1,NLER)
  197 FORMAT (20F4.2)
  195 CONTINUE


  190 CONTINUE
```

```
      WRITE(6,14)
   14 FORMAT ('0',49X,'STUDENT RECORDS')
      WRITE(6,17)
   17 FORMAT('0', 8X,'STUDENT NAME',9X,' VL  VN  AC  AN  KT  SI  SO',
     1'  EO  EW   SO   SH  SEX',' 1 2 3 4 5 6 7 8 91011121314'/)

C ---------------------------------------------------------------------
C     READ IN STUDENT RECORDS
      DO 15 I=1,NTOT
      READ(5,12)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
   12 FORMAT(I4,1X,4A6,2X,9F2.0,F3.1,1X,F2.0,F1.0,1X,14I1)
      WRITE(6,13)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
   13 FORMAT(I5,1X,4A6,9F4.0,F5.1,1X,F4.0,2X,F2.0,14I2)
   15 CONTINUE


C ---------------------------------------------------------------------
C     LEFT-JUSTIFY SELECTED SKILLS AND LEARNER VARIABLES
      DO 250 I=1,NTOT
      DO 220 II=1,NLER
      KK=NLVNOS(II)
      IF (L.EQ.0) VLEARN(KK,I)=0.0
  220 VLEARN(II,I)=VLEARN(KK,I)
      DO 240 II=1,NSKILL
      KK=NSKNOS(II)
      IF (K.EQ.0) ISKILL(KK,I)=1
  240 ISKILL(II,I)=ISKILL(KK,I)
  250 CONTINUE


C ---------------------------------------------------------------------
C     CALCULATE NUMBER OF STUDENTS ELIGIBLE FOR EACH SKILL
      WRITE (6,130)
  130 FORMAT ('0NUMBER OF STUDENTS ELIGIBLE FOR THE FOLLOWING SKILLS -
      DO 140 J=1,NSKILL
      NUM(J)=0
      DO 150 I=1,NTOT
      NUM(J)=NUM(J)+ISKILL(J,I)
  150 CONTINUE
      WRITE(6,160)(SKNAME(II,J),II=1,3),NUM(J)
  160 FORMAT (1X,3A6,' -- ',I3)
  140 CONTINUE


C ---------------------------------------------------------------------
C     CALCULATE MEAN, VARIANCE, STD. DEV. OF EACH LEARNER VARIABLE
      WRITE (6,70)
   70 FORMAT ('0 *** LEARNER VARIABLES - MEAN, VAR. AND STD. DEV. ***'
      DO 90 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 80 J=1,NTOT
      IF(VLEARN(I,J).LT.0)GO TO 80
      SUM=SUM+VLEARN(I,J)
      SUMSQ=VLEARN(I,J)**2+SUMSQ
      XN=XN+1.
   80 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
   90 WRITE(6,100)(LVNAME(J,I),J=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  100 FORMAT(' ',3A6,' HAS MEAN = ',F6.2,2X,'VAR. = ',F8.2,2X,'SD. = '
     1,F6.2)


C -----------------------------------------------------------------
C     MISSING DATA
      DO 200 I=1,NTOT
      DO 210 J=1,NLER
      IF(VLEARN(J,I).GE.0)GO TO 210
      VLEARN(J,I)=XMEAN(J)
  210 CONTINUE
  200 CONTINUE


C -----------------------------------------------------------------
C     STANDARDIZE STUDENT VARIABLE SCORES
      DO 95 J=1,NTOT
      DO 97 I=1,NLER
      IF (XSTDEV(I).LT. 0.0001) GOTO 98
      SLEARN(I,J)=(VLEARN(I,J)-XMEAN(I))/XSTDEV(I)
      GOTO 97
   98 SLEARN(I,J)=XMEAN(I)
   97 CONTINUE
   95 CONTINUE

      WRITE (6,214)
  214 FORMAT ('0 *** STANDARDIZED LEARNER VARIABLES ***')
      DO 215 I=1,NTOT
  215 WRITE (6,216)  ID(I),(XNAME(J,I),J=1,4),(SLEARN(J,I),J=1,NLER)
  216 FORMAT (I5,1X,4A6,14(F6.2))


C -----------------------------------------------------------------
C     PARTITION GROUPS BASED ON SHORTEST DISTANCE
      IF (N.EQ.1) CALL SEEDY


C -----------------------------------------------------------------
C     OUTPUT GROUP MEANS
```

```
      PDIST=0.0
      KKK=0
  450 CONTINUE
      WRITE (6,410)
  410 FORMAT ('0 *** MEAN OF EACH GROUP ***')
      DO 420 I=1,NG
  420 WRITE (6,430) I,(SEED(J,I),J=1,NLER)
  430 FORMAT(' GROUP #',I2,13(F7.3))


C ----------------------------------------------------------------------
C     ASSIGN STUDENTS TO SEED POINTS
      TDIST=0.0
      TSSW=0.0
      WRITE (6,470)
  470  FORMAT ('0 *** STUDENT GROUP ASSIGNMENTS ***')
      DO 500 I=1,NTOT
      NSTUNO(I)=0
      DO 490 J=1,NG
      X=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (NSTUNO(I).EQ.0) GOTO 480
      IF (X.GT.XDIST) GOTO 490
  480 XDIST=X
      NSTUNO(I)=J
  490 CONTINUE
      IF (NSTUNO(I).EQ.0) XDIST=0.0
      SDIST(I)=XDIST
      TDIST=TDIST+XDIST
      TSSW=TSSW+XDIST**2
  500 CONTINUE
      WRITE (6,495) (I,NSTUNO(I),SDIST(I),I=1,NTOT)
  495 FORMAT (3(7X,'STU #',I3,' ASG TO ',I2,' W. DIST ',F5.2))


C ----------------------------------------------------------------------
C     COMPUTE MEAN AND VAR. OF ALL GROUPS
      DO 600 J=1,NG
      DO 590 I=1,NLER
      SUM=0.0
      XN=0.0
      DO 580 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 580
      SUM=SUM+SLEARN(I,II)
      XN=XN+1.
  580 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      SEED(I,J)=SUM/XN
  590 CONTINUE
  600 CONTINUE

      IF (KKK.EQ.0) V=TDIST
      IF (KKK.NE.0) V=ABS(TDIST-PDIST)
      KKK=KKK+1
      POIST=TDIST
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,620) KKK,TDIST,V,TSSW,XMSSW
  620 FORMAT ('0FOR ITERATION #',I2,' TOTAL DISTANCE = ',F7.3,
     1' WHICH DIFFERS FROM PREVIOUS BY ',F7.3,
     2 /,5X,'TOTAL SUM OF SQUARES WITHIN = ',F8.2,
     3   ',  MEAN SUM OF SQUARES WITHIN = ',F7.2)
      IF (V.GT.0.001 .AND. KKK.LE.ILIMIT) GOTO 450


C ----------------------------------------------------------------------
C     OUTPUT NUMBER ELIGIBLE IN EACH GROUP
      WRITE (6,305) (I,I=1,NSKILL)
  305 FORMAT ('0GRP # ',15I4)
      DO 330 J=1,NG
      DO 320 I=1,NSKILL
      NGELIG(I,J)=0
      DO 310 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 310
      IF (ISKILL(I,II).EQ.0) GOTO 310
      NGELIG(I,J)=NGELIG(I,J)+1
  310 CONTINUE
  320 CONTINUE
      WRITE (6,325) J,(NGELIG(I,J),I=1,NSKILL)
  325 FORMAT (2X,I2,3X,15I4)
  330 CONTINUE


C ----------------------------------------------------------------------
C     ASSIGN SKILLS TO GROUPS
      DO 335 I=1,NG
  335 NGSKIL(I)=0
      KKK=0
  340 CONTINUE
      MAX=0
      DO 370 J=1,NG
      IF (NGSKIL(J).NE.0) GOTO 370
      DO 360 I=1,NSKILL
      IF (M.EQ.1) GOTO 355
      DO 350 II=1,NG
      IF (NGSKIL(II).EQ.I) GOTO 360
  350 CONTINUE
  355 IF (NGELIG(I,J).LE.MAX) GOTO 360
      MAX=NGELIG(I,J)
      IG=J
      IS=I
  360 CONTINUE
  370 CONTINUE
      NGSKIL(IG)=IS
      KKK=KKK+1
      IF (KKK.LT.NG) GOTO 340
```

```
C ----------------------------------------------------------------------
C     OUTPUT OMISSIONS AND NO. IN EACH GROUP
      WRITE (6,625)
  625 FORMAT ('1',' *** STUDENT OMISSIONS ***',/)
      DO 630 J=1,NG
  630 NINGRP(J)=0
      DO 640 I=1,NTOT
      JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 635
      NINGRP(JJ)=NINGRP(JJ)+1
      GOTO 640
  635 WRITE (6,13) ID(I),(XNAME(II,I),II=1,4)
  640 CONTINUE
      WRITE (6,655)
  655 FORMAT('0GRP #   SKILL   NO. IN GROUP')
      WRITE (6,660) (J,NGSKIL(J),NINGRP(J),J=1,NG)
  660 FORMAT (I4,4X,I4,8X,I4)


C ----------------------------------------------------------------------
C     CHECK FOR STUDENTS NOT ELIGILBE FOR SKILL OF GROUP
      WRITE (6,963)
  963 FORMAT ('0REMOVE INELIGIBLE STUDENTS FROM GROUPS',/)
      ASSIGN 960 TO IRETN
      INCOM=.FALSE.
      SIZEC=.FALSE.
  960 CONTINUE
      IX=0
      X=0.0
      DO 970 I=1,NTOT
      JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 970
      II=NGSKIL(JJ)
      IF (ISKILL(II,I).EQ.1) GOTO 970
      SDIST(I)=DIST(SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 970
      X=SDIST(I)
      IX=I
  970 CONTINUE
      IF (IX.NE.0) GOTO 690
      SIZEC=.TRUE.
      WRITE (6,990)
  980 FORMAT ('0PROCESS SIZE CONSTRAINTS',/)

C ----------------------------------------------------------------------
C     CHECK FOR GROUPS OVERLOADED
      ASSIGN 670 TO IRETN
      INCOM=.FALSE.
  670 CONTINUE
      DO 675 J=1,NG
      IF (NINGRP(J).GT.NHIGH(J)) GOTO 677
  675 CONTINUE
      GOTO 800


C ------------------------------------------------------------------
C     COMPUTE DISTANCES AND FIND FARTHEST STUDENT IN OVERLOADED GROUP
  677 CONTINUE
      X=0.0
      IX=0
      DO 685 J=1,NG
      IF (NINGRP(J).LE.NHIGH(J)) GOTO 685
      DO 680 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 680
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 680
      X=SDIST(I)
      IX=I
  680 CONTINUE
  685 CONTINUE


C ------------------------------------------------------------------
C     FIND ANOTHER GROUP FOR THIS STUDENT
  690 CONTINUE
      Y=0.0
      IY=0
      J=NSTUNO(IX)
      DO 700 JJ=1,NG
      IF (J.EQ.JJ) GOTO 700
      IF (SIZEC .AND. NINGRP(JJ).GE.NHIGH(JJ)) GOTO 700
      NSK=NGSKIL(JJ)
      IF (ISKILL(NSK,IX).EQ.0) GOTO 700

C ---------- CHECK FOR INCOMPATIBLES IN SAME GROUP
      IF (.NOT.INCOM) GOTO 698
      DO 695 II=1,NIC
      I=NICNOS(II)
      IF (I.EQ.IX) GOTO 695
      IF (NSTUNO(I).EQ.JJ) GOTO 700
  695 CONTINUE
  698 CONTINUE
      YY=DIST (SLEARN(1,IX),SEED(1,JJ),NLER)
      IF (IY.NE.0 .AND. YY.GT.Y) GOTO 700
      IY=JJ
      Y=YY
  700 CONTINUE
      IF (IY.NE.0) GOTO 750


C ------------------------------------------------------------------
C     CANNOT MOVE - SO PUT IN OMISSIONS
```

```
      WRITE (6,720) IX,J
  720 FORMAT ('0 STUDENT #',I3,' HAS BEEN BOOTED OUT OF GRP #',I2)
      WRITE (6,710) ID(IX),(XNAME(II,IX),II=1,4),J
  710 FORMAT (I5,1X,4A6,' HAS BEEN REMOVED FROM GRP #',I2)
      NSTUNO(IX)=0
      SDIST(IX)=0.0
      NINGRP(J)=NINGRP(J)-1
      GOTO 760


C -----------------------------------------------------------------------
C      MOVE STUDENT AND RECOMPUTE GROUP MEANS
  750 CONTINUE
      WRITE (6,755) IX,J,IY
  755 FORMAT ('0 STUDENT #',I3,' IS TO BE MOVED FROM GRP #',
     1  I2,' TO GRP #',I2)
      NSTUNO(IX)=IY
      NINGRP(J)=NINGRP(J)-1
      NINGRP(IY)=NINGRP(IY)+1

      DO 790 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 780 I=1,NTOT
      IF (NSTUNO(I).NE.IY) GOTO 780
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  780 CONTINUE
      SEED(JJ,IY)=SUM/XN
  790 CONTINUE

  760 CONTINUE
      DO 775 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 770 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 770
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  770 CONTINUE
      SEED(JJ,J)=SUM/XN
  775 CONTINUE


C -----------------------------------------------------------------------
C      RECOMPUTE STUDENT DISTANCES
      TDIST=0.0
      TSSW=0.0
      DO 740 I=1,NTOT
      SDIST(I)=0.0
      J=NSTUNO(I)
      IF (J.EQ.0) GOTO 740
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      X=SDIST(I)
      TDIST=TDIST+X
      TSSW=TSSW+X**2
      SDIST(I)=X
  740 CONTINUE
      XMSSW=TSSW/FLOAT(NTOT-NO)
      WRITE (6,745) TDIST,TSSW,XMSSW
  745 FORMAT(5X,'TOTAL DIST. = ',F7.3,' TSSW = ',F8.2,' MSSW = ',F7.2)
      GOTO IRETN


C ----------------------------------------------------------------------
C     PROCESS INCOMPATIBLE STUDENTS IN SIMILAR MANNER
  800 CONTINUE
      INCOM=.TRUE.
      READ (5,810,END=900) NIC,NICNOS
  810 FORMAT (I2,10I4)
      WRITE (6,815) (NICNOS(I),I=1,NIC)
  815 FORMAT ('0INCOMPATIBLE STUDENTS = ',10I5)
      IF (NIC.LE.1) GOTO 800
      DO 830 I=1,NIC
      DO 820 J=1,NTOT
      IF (ID(J).EQ.NICNOS(I)) GOTO825
  820 CONTINUE
      WRITE (6,822) NICNOS(I)
  822 FORMAT (' NO STUDENT FOR ID #',I4)
      GOTO 800
  825 NICNOS(I)=J
  830 CONTINUE

  860 CONTINUE
      DO 840 J=1,NO
      KOUNT=0
      DO 835 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 835
      KOUNT=KOUNT+1
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
  835 CONTINUE
      IF (KOUNT.LE.1) GOTO 840

      X=0.0
      IX=0
      DO 850 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 850
      IF (SDIST(I).LT.X) GOTO 850
      X=SDIST(I)
      IX=I
  850 CONTINUE
      ASSIGN 860 TO IRETN
      GOTO 690
  840 CONTINUE
      GOTO 800


C -----------------------------------------------------------------------
C     OUTPUT FINAL TOTALS AND LEAVE
  900 CONTINUE
      DO 950 J=1,NG
      NSK=NGSKIL(J)
      WRITE (6,910) J,NSK,(SKNAME(JJ,NSK),JJ=1,3),NINGRP(J)
  910 FORMAT ('1','GROUP #',I2,', SKILL #',I2,2X,3A6,/,
     1' NUMBER OF STUDENTS RECOMMENDED = ',I3,//,
     2 ' STUD #',12X,'STUDENT NAME',10X,'DISTANCE',/)
      DO 940 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 930 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 930
      SUM=SUM+SLEARN(I,II)
      SUMSQ=SLEARN(I,II)**2+SUMSQ
      XN=XN+1.0
      IF (I.NE.1) GOTO 930
      SDIST(II)=DIST(SLEARN(1,II),SFED(1,J),NLER)
      WRITE (6,920) ID(II),(XNAME(JJ,II),JJ=1,4),SDIST(II)
  920 FORMAT (2X,I4,4X,4A6, 5X,F7.3)
  930 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
      IF (I.EQ.1) WRITE (6,70)
      WRITE (6,100) (LVNAME(JJ,I),JJ=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  940 CONTINUE
  950 CONTINUE
      CALL EXIT
      END
```

```
C---- SEED POINT ROUTINE - ALGORITHM B ----

      SUBROUTINE SEEDY
      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=15
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     +       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     +       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION NPART (MAXGRP)


C -------------------------------------------------------------------
C     FIND 2-STUDENTS W. GREATEST PAIR WISE DISTANCE
      NSTU1=1
      NSTU2=1
      DMAX=0.0
      DO 10 I=1,NTOT
      NN=I+1
      DO 15 II=NN,NTOT
      X=DIST (SLEARN(1,I),SLEARN(1,II),NLER)
      IF (X.LE.DMAX) GOTO 15
      NSTU1=I
      NSTU2=II
      DMAX=X
15    CONTINUE
10    CONTINUE
      WRITE (6,17) NSTU1,NSTU2,DMAX
   17 FORMAT('0LARGEST DIST!  1ST STUDENT = ',I3,', 2ND STUDENT = ',
     1 I3,', DIFFERENCE = ',F9.3)


C -------------------------------------------------------------------
C     CONSTRUCT A TABLE OF STUDENT DISTANCES
      NELIG=0
      DO 100 I=1,NTOT
      X=DIST (SLEARN (1,I), SLEARN(1,NSTU1),NLER)
      IF (NELIG.EQ.0) GOTO 30
      DO 20 II=1,NELIG
      IF (X.LT.SDIST(II)) GOTO 40
20    CONTINUE
30    SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=I
      GOTO 90
40    NSTU2=I
      DO 50 III=II,NELIG
      TEMP=SDIST(III)
      SDIST (III)=X
      X=TEMP
      NTEMP=NSTUNO(III)
      NSTUNO(III)=NSTU2
50    NSTU2=NTEMP
      SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=NSTU2
90    NELIG=NELIG+1
100   CONTINUE
      WRITE (6,80) (SDIST(I),NSTUNO(I),I=1,NELIG)
   80 FORMAT (5(2X,F9.3,2X,I3))


C ----------------------------------------------------------------------
C     SUM THE LOWER GROUP LIMITS
      SUM=0.0
      DO 110 I=1,NG
      SUM=SUM+FLOAT(NLOW(I))
110   CONTINUE


C ----------------------------------------------------------------------
C     ESTABLISH A PARTITIONING
      NEQSK=0
      DO 120 I=1,NG
      NEQSK=NEQSK+1
      PORTN=(FLOAT(NLOW(I))/SUM)*FLOAT(NELIG)
      IPORTN=PORTN
      IF((PORTN-IPORTN).GE.0.5) IPORTN=IPORTN+1
      NPART(NEQSK)=IPORTN
120   CONTINUE


C ----------------------------------------------------------------------
C     BALANCE THE PARTITIONS
      WRITE (6,135)
  135 FORMAT ('0 *** GROUP PARTITIONING ***')
      WRITE (6,145) NELIG,(NPART(J),J=1,NEQSK)
145   FORMAT (' COMPUTED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      DO 140 J=1,NEQSK
      NSUM=0
      DO 130 I=1,NEQSK
130   NSUM=NSUM+NPART(I)
      IF(NSUM.EQ.NELIG) GOTO 150
      IF(NSUM.GT.NELIG) NPART(J)=NPART(J)-1
      IF(NSUM.LT.NELIG) NPART(J)=NPART(J)+1
140   CONTINUE
150   CONTINUE
      WRITE (6,155) NELIG,(NPART(J),J=1,NEQSK)
155   FORMAT (' BALANCED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      IF(NSUM.NE.NELIG) STOP


C ----------------------------------------------------------------------
C     SELECT MID-POINTS
      I=(NELIG+1)/2
      II=(NELIG+2)/2
      X1=SDIST(I)-SDIST(1)
      X2=SDIST(NELIG)-SDIST(II)


C ------------------------------------------------------------------------
C     FIND MEAN OF EACH PARTITION
      NEQSK=0
      II=0
      DO 200 I=1,NG
      NEQSK=NEQSK+1
      NN=NPART(NEQSK)+II
      N=II+1
      N2=NN
      N1=N
      IF(X1.LT.X2) GOTO 160
      N2=NELIG+1-N
      N1=NELIG+1-NN
160   DO 190 K=1,NLER
      XN=0.
      SUM=0.
      DO 180 J=N1,N2
      JJ=NSTUNO(J)
      SUM=SUM+SLEARN(K,JJ)
180   XN=XN+1.
190   SEED(K,I)=SUM/XN
      II=NN
200   CONTINUE
      RETURN
      END
```

APPENDIX C

COMPUTER PROGRAM FOR GROUPAL C

```
C---- GROUPING ALGORITHM C ----

      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=30
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     *       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT,WEIGHT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     *       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION XNAME(4,MAXSTU),VLEARN(MAXLV,MAXSTU)
      DIMENSION ID(MAXSTU)
      DIMENSION XMEAN(MAXLV),XSTDEV(MAXLV),XVAR(MAXLV)
      DIMENSION NUM(MAXSK)
      DIMENSION NSKNOS(MAXSK),NLVNOS(MAXLV)
      INTEGER   SKNAME(3,MAXSK),LVNAME(3,MAXLV)
      DIMENSION NINGRP(MAXGRP)
      DIMENSION NICNOS(10)
      LOGICAL INCOM


C -----------------------------------------------------------------
C     INPUT AND ECHO PARAMETER CARDS
      WRITE(6,1)
    1 FORMAT('1','GROUPING ALGORITHM C',/,1X,20('-'),/)

      READ(5,2)NG,NSKILL,NLER,NTOT,WEIGHT,ILIMIT
    2 FORMAT(I2,1X,I2,1X,I2,1X,I3,1X,F4.2,1X,I2)
      WRITE(6,3)NG,NSKILL,NLER,NTOT,WEIGHT
    3 FORMAT('0NUMBER OF GROUPS REQUESTED = ',I2,/,
     1       ' NUMBER OF SKILLS CONSIDERED = ',I2,/,
     2       ' NUMBER OF LEARNER VARIABLES CONSIDERED = ',I2,/,
     3       ' NUMBER OF STUDENTS = ',I3,/,
     4       ' WEIGHT APPLIED TO SKILLS = ',F5.2,/)
      IF (ILIMIT.EQ.0) ILIMIT=10

      READ(5,4)K,L,M
    4 FORMAT(3I1)
      IF(K.EQ.1)WRITE(6,5)
    5 FORMAT(' ELIGIBILITY FOR SKILLS TAKEN INTO ACCOUNT')
      IF(K.EQ.0)WRITE(6,6)
    6 FORMAT(' ELIGIBILITY FOR SKILLS NOT TAKEN INTO ACCOUNT')
      IF(L.EQ.1)WRITE(6,7)
    7 FORMAT(' LEARNER VARIABLES TAKEN INTO ACCOUNT')
      IF(L.EQ.0)WRITE(6,8)
    8 FORMAT(' LEARNER VARIABLES NOT TAKEN INTO ACCOUNT')
      IF(M.EQ.1)WRITE(6,9)
    9 FORMAT(' MORE THAN ONE GROUP MAY STUDY THE SAME SKILL')
      IF(M.EQ.0)WRITE(6,11)
   11 FORMAT(' ONLY ONE GROUP MAY STUDY A PARTICULAR SKILL')

      READ(5,10)(NLOW(I),NHIGH(I),I=1,NG)
   10 FORMAT (20I2)
      WRITE (6,19) (NLOW(I),NHIGH(I),I=1,NG)
   19 FORMAT('0GROUP RANGES = ',10(2I3,2X))
```

```



C ------------------------------------------------------------------------
C      RANGE TEST
       MSUM=0
       NSUM=0
       KLOW=99
       DO 20 I=1,NG
       NSUM=NSUM+NLOW(I)
       MSUM=MSUM+NHIGH(I)
       IF (NLOW(I).GT.NHIGH(I)) WRITE (6,16) I,NLOW(I),NHIGH(I)
    16 FORMAT (' INVALID RANGE  GRP #',I2,' LOW = ',I2,', HIGH = ',I2)
       IF (NLOW(I).GT.KLOW) WRITE (6,18) I,NLOW(I),KLOW
    18 FORMAT (' SEQUENCE ERROR  GRP #',I2,' CURRENT LOW = ',
      1 I2,', PREVIOUS LOW = ',I2)
       KLOW = NLOW(I)
    20 CONTINUE
       IF(NTOT.GT.MSUM.OR.NTOT.LT.NSUM)WRITE(6,30)NTOT
    30 FORMAT(' RANGE SIZES DO NOT CORRESPOND TO TOTAL NUMBER OF STUDEN
      1',I3)


C ------------------------------------------------------------------------
C      READ IN SKILL AND LEARNER VARIABLE NAMES
       READ (5,10) (NSKNOS(I),I=1,NSKILL)
       WRITE (6,185) (NSKNOS(I),I=1,NSKILL)
   185 FORMAT ('0SKILLS CONSIDERED = ',20I3)
       READ (5,10) (NLVNOS(I),I=1,NLER)
       WRITE (6,187) (NLVNOS(I),I=1,NLER)
   187 FORMAT ('0LEARNER VARIABLES CONSIDERED = ',20I3)
       DO 170 I=1,NSKILL
       READ(5,180)(SKNAME(J,I),J=1,3)
   180 FORMAT(3A6)
   170 CONTINUE
       DO 175 I=1,NLFR
       READ(5,180)(LVNAME(J,I),J=1,3)
   175 CONTINUE


       WRITE(6,14)
    14 FORMAT ('0',49X,'STUDENT RECORDS')
       WRITE(6,17)
    17 FORMAT('0', 8X,'STUDENT NAME',9X,' VL  VN  AC  AN  KT  SI  SO',
      1'  EO  EW   SD   SM  SEX',' 1 2 3 4 5 6 7 8 91011121314'/)

C ------------------------------------------------------------------------
C      READ IN STUDENT RECORDS
       DO 15 I=1,NTOT
       READ(5,12)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
      1(ISKILL(J,I),J=1,14)
    12 FORMAT(I4,1X,4A6,2X,9F2.0,F3.1,1X,F2.0,F1.0,1X,14I1)
       WRITE(6,13)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
   13 FORMAT(I5,1X,4A6,9F4.0,F5.1,1X,F4.0,2X,F2.0,14I2)
   15 CONTINUE


C -----------------------------------------------------------------------
C     LEFT-JUSTIFY SELECTED SKILLS AND LEARNER VARIABLES
      DO 250 I=1,NTOT
      DO 220 II=1,NLER
      KK=NLVNOS(II)
      IF (L.EQ.0) VLEARN(KK,I)=0.0
  220 VLEARN(II,I)=VLEARN(KK,I)
      JJ=NLER
      DO 240 II=1,NSKILL
      KK=NSKNOS(II)
      IF (K.EQ.0) ISKILL(KK,I)=1
      ISKILL(II,I)=ISKILL(KK,I)
      JJ=JJ+1
      VLEARN(JJ,I)=FLOAT(ISKILL(KK,I))
      DO 230 KK=1,3
  230 LVNAME(KK,JJ)=SKNAME(KK,II)
  240 CONTINUE
  250 CONTINUE
      NLER=NLER+NSKILL


C -----------------------------------------------------------------------
C     CALCULATE NUMBER OF STUDENTS ELIGIBLE FOR EACH SKILL
      WRITE (6,130)
  130 FORMAT ('0NUMBER OF STUDENTS ELIGIBLE FOR THE FOLLOWING SKILLS -
      DO 140 J=1,NSKILL
      NUM(J)=0
      DO 150 I=1,NTOT
      NUM(J)=NUM(J)+ISKILL(J,I)
  150 CONTINUE
      WRITE(6,160)(SKNAME(II,J),II=1,3),NUM(J)
  160 FORMAT (1X,3A6,' -- ',I3)
  140 CONTINUE


C -----------------------------------------------------------------------
C     CALCULATE MEAN, VARIANCE, STD. DEV. OF EACH LEARNER VARIABLE
      WRITE (6,70)
   70 FORMAT ('0 *** LEARNER VARIABLES - MEAN, VAR. AND STD. DEV. ***'
      DO 90 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 80 J=1,NTOT
      IF(VLEARN(I,J).LT.0)GO TO 80
      SUM=SUM+VLEARN(I,J)
      SUMSQ=VLEARN(I,J)**2+SUMSQ
      XN=XN+1.
   80 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
   90 WRITE(6,100)(LVNAME(J,I),J=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  100 FORMAT(' ',3A6,' HAS MEAN = ',F6.2,2X,'VAR. = ',F8.2,2X,'SD. = '
     1,F6.2)


C -----------------------------------------------------------------
C     MISSING DATA
      DO 200 I=1,NTOT
      DO 210 J=1,NLER
      IF(VLEARN(J,I).GE.0)GO TO 210
      VLEARN(J,I)=XMEAN(J)
  210 CONTINUE
  200 CONTINUE


C -----------------------------------------------------------------
C     STANDARDIZE STUDENT VARIABLE SCORES
      DO 95 J=1,NTOT
      DO 97 I=1,NLER
      IF (XSTDEV(I).LT. 0.0001) GOTO 98
      SLEARN(I,J)=(VLEARN(I,J)-XMEAN(I))/XSTDEV(I)
      GOTO 97
   98 SLEARN(I,J)=XMEAN(I)
   97 CONTINUE
   95 CONTINUE

      WRITE (6,214)
  214 FORMAT ('0 *** STANDARDIZED LEARNER VARIABLES ***')
      DO 215 I=1,NTOT
  215 WRITE (6,216)  ID(I),(XNAME(J,I),J=1,4),(SLEARN(J,I),J=1,NLER)
  216 FORMAT (I5,1X,4A6,14(F6.2))


C -----------------------------------------------------------------
C     ALLOCATE SKILLS TO GROUP BASED ON LARGEST NO. ELIGIBLE
      DO 350 I=1,NG

      MAX=0
      DO 310 II=1,NSKILL
      IF (NUM(II).LE.MAX) GOTO 310
      NSK=II
      MAX=NUM(II)
  310 CONTINUE

      IF (MAX.GE.NLOW(I)) GOTO 330
      WRITE (6,320) I,NLOW(I),MAX
  320 FORMAT('0LOWER LIMIT OF GROUP #',I2,' = ',I2,
     +       ' IS GREATER THAN # OF STUDENTS ELIGIBLE = ',I3)
      NLOW(I)=MAX
  330 NUM(NSK)=MAX-NLOW(I)
      NGSKIL(I)=NSK
      IF (M.EQ.0) NUM(NSK)=0
  350 CONTINUE
      WRITE (6,351) (NGSKIL(I),I=1,NG)
  351 FORMAT ('0SKILLS ASSIGNED TO GROUPS - ',20I3)


C -------------------------------------------------------------------
C     CALCULATE SEED POINT FOR EACH GROUP
      DO 400 I=1,NSKILL

      KOUNT=0
      DO 360 II=1,NG
      IF (NGSKIL(II).NE.I) GOTO 360
      KOUNT=KOUNT+1
      NGNO=II
  360 CONTINUE

      IF (KOUNT.EQ.0) GOTO 400
      IF (KOUNT.EQ.1) GOTO 370
      CALL SEEDY(I)
      GOTO 400

  370 DO 390 J=1,NLER
      XN=0.
      SUM=0.
      DO 380 II=1,NTOT
      IF (ISKILL(I,II).EQ.0) GOTO 380
      XN=XN+1.
      SUM=SUM+SLEARN(J,II)
  380 CONTINUE
  390 SEED(J,NGNO)=SUM/XN
  400 CONTINUE


C -------------------------------------------------------------------
C     OUTPUT GROUP MEANS
      PDIST=0.0
      KKK=0
  450 CONTINUE
      WRITE (6,410)
  410 FORMAT ('0 *** MEAN OF EACH GROUP ***')
      DO 420 I=1,NG
  420 WRITE (6,430) I,(SEED(J,I),J=1,NLER)
  430 FORMAT(' GROUP #',I2,13(F7.3))


C -------------------------------------------------------------------
```

```
C     ASSIGN STUDENTS TO SEED POINTS
      TDIST=0.0
      TSSW=0.0
      WRITE (6,470)
 470  FORMAT ('0 *** STUDENT GROUP ASSIGNMENTS ***')
      DO 500 I=1,NTOT
      NSTUNO(I)=0
      DO 490 J=1,NG
      NSK=NGSKIL(J)
      IF (ISKILL(NSK,I).EQ.0) GOTO 490
      X=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (NSTUNO(I).EQ.0) GOTO 480
      IF (X.GT.XDIST) GOTO 490
  480 XDIST=X
      NSTUNO(I)=J
  490 CONTINUE
      IF (NSTUNO(I).EQ.0) XDIST=0.0
      SDIST(I)=XDIST
      J=NSTUNO(I)
      IF (J.NE.0) XDIST = DIST (SLEARN(1,I),SEED(1,J),NLER-NSKILL)
      TDIST=TDIST+XDIST
      TSSW=TSSW+XDIST**2
  500 CONTINUE
      WRITE (6,495) (I,NSTUNO(I),SDIST(I),I=1,NTOT)
  495 FORMAT (3(7X,'STU #',I3,' ASG TO ',I2,' W. DIST ',F5.2))


C ----------------------------------------------------------------------
C     COMPUTE MEAN AND VAR. OF ALL GROUPS
      DO 600 J=1,NG
      DO 590 I=1,NLER
      SUM=0.0
      XN=0.0
      DO 580 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 580
      SUM=SUM+SLEARN(I,II)
      XN=XN+1.
  580 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      SEED(I,J)=SUM/XN
  590 CONTINUE
  600 CONTINUE

      IF (KKK.EQ.0) V=TDIST
      IF (KKK.NE.0) V=ABS(TDIST-PDIST)
      KKK=KKK+1
      PDIST=TDIST
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,620) KKK,TDIST,V,TSSW,XMSSW
  620 FORMAT ('0FOR ITERATION #',I2,' TOTAL DISTANCE = ',F7.3,
     1' WHICH DIFFERS FROM PREVIOUS BY ',F7.3,
     2 /,5X,'TOTAL SUM OF SQUARES WITHIN = ',F8.2,
     3   ',  MEAN SUM OF SQUARES WITHIN = ',F7.2)
      IF (V.GT.0.001 .AND. KKK.LE.ILIMIT) GOTO 450


C ----------------------------------------------------------------------
C     OUTPUT OMISSIONS AND NO. IN EACH GROUP
      WRITE (6,625)
  625 FORMAT ('1',' *** STUDENT OMISSIONS ***',/)
      DO 630 J=1,NG
  630 NINGRP(J)=0
      DO 640 I=1,NTOT
      JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 635
      NINGRP(JJ)=NINGRP(JJ)+1
      GOTO 640
  635 WRITE (6,13) ID(I),(XNAME(II,I),II=1,4)
  640 CONTINUE
      WRITE (6,655)
  655 FORMAT('0GRP #   SKILL   NO. IN GROUP')
      WRITE (6,660) (J,NGSKIL(J),NINGRP(J),J=1,NG)
  660 FORMAT (I4,4X,I4,8X,I4)

C ----------------------------------------------------------------------
C     CHECK FOR GROUPS OVERLOADED
      ASSIGN 670 TO IRETN
      INCOM=.FALSE.
  670 CONTINUE
      DO 675 J=1,NG
      IF (NINGRP(J).GT.NHIGH(J)) GOTO 677
  675 CONTINUE
      GOTO 800


C ----------------------------------------------------------------------
C     COMPUTE DISTANCES AND FIND FARTHEST STUDENT IN OVERLOADED GROUP
  677 CONTINUE
      X=0.0
      IX=0
      DO 685 J=1,NG
      IF (NINGRP(J).LE.NHIGH(J)) GOTO 685
      DO 680 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 680
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 680
      X=SDIST(I)
      IX=I
  680 CONTINUE
  685 CONTINUE


C ----------------------------------------------------------------------
C     FIND ANOTHER GROUP FOR THIS STUDENT
```

```
  690 CONTINUE
      Y=0.0
      IY=0
      J=NSTUNO(IX)
      DO 700 JJ=1,NG
      IF (J.EQ.JJ) GOTO 700
      IF (NINGRP(JJ).GE.NHIGH(JJ)) GOTO 700
      NSK=NGSKIL(JJ)
      IF (ISKILL(NSK,IX).EQ.0) GOTO 700

C ---------- CHECK FOR INCOMPATIBLES IN SAME GROUP
      IF (.NOT.INCOM) GOTO 698
      DO 695 II=1,NIC
      I=NICNOS(II)
      IF (I.EQ.IX) GOTO 695
      IF (NSTUNO(I).EQ.JJ) GOTO 700
  695 CONTINUE
  698 CONTINUE
      YY=DIST (SLEARN(1,IX),SEED(1,JJ),NLER)
      IF (IY.NE.0 .AND. YY.GT.Y) GOTO 700
      IY=JJ
      Y=YY
  700 CONTINUE
      IF (IY.NE.0) GOTO 750


C ----------------------------------------------------------------------
C     CANNOT MOVE - SO PUT IN OMISSIONS
      WRITE (6,720) IX,J
  720 FORMAT ('0 STUDENT #',I3,' HAS BEEN BOOTED OUT OF GRP #',I2)
      WRITE (6,710) ID(IX),(XNAME(II,IX),II=1,4),J
  710 FORMAT (I5,1X,4A6,' HAS BEEN REMOVED FROM GRP #',I2)
      NSTUNO(IX)=0
      SDIST(IX)=0.0
      NINGRP(J)=NINGRP(J)-1
      GOTO 760


C ----------------------------------------------------------------------
C     MOVE STUDENT AND RECOMPUTE GROUP MEANS
  750 CONTINUE
      WRITE (6,755) IX,J,IY
  755 FORMAT ('0 STUDENT #',I3,' IS TO BE MOVED FROM GRP #',
     1  I2,' TO GRP #',I2)
      NSTUNO(IX)=IY
      NINGRP(J)=NINGRP(J)-1
      NINGRP(IY)=NINGRP(IY)+1

      DO 790 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 780 I=1,NTOT
      IF (NSTUNO(I).NE.IY) GOTO 780
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  780 CONTINUE
      SEED(JJ,IY)=SUM/XN
  790 CONTINUE

  760 CONTINUE
      DO 775 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 770 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 770
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  770 CONTINUE
      SEED(JJ,J)=SUM/XN
  775 CONTINUE


C --------------------------------------------------------------------
C     RECOMPUTE STUDENT DISTANCES
      TDIST=0.0
      TSSW=0.0
      DO 740 I=1,NTOT
      SDIST(I)=0.0
      J=NSTUNO(I)
      IF (J.EQ.0) GOTO 740
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      X = DIST (SLEARN(1,I),SEED(1,J),NLER-NSKILL)
      TDIST=TDIST+X
      TSSW=TSSW+X**2
      SDIST(I)=X
  740 CONTINUE
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,745) TDIST,TSSW,XMSSW
  745 FORMAT(5X,'TOTAL DIST. = ',F7.3,' TSSW = ',F8.2,' MSSW = ',F7.2)
      GOTO IRETN


C --------------------------------------------------------------------
C     PROCESS INCOMPATIBLE STUDENTS IN SIMILAR MANNER
  800 CONTINUE
      INCOM=.TRUE.
      READ (5,810,END=900) NIC,NICNOS
  810 FORMAT (I2,10I4)
      WRITE (6,815) (NICNOS(I),I=1,NIC)
  815 FORMAT ('0INCOMPATIBLE STUDENTS = ',10I5)
      IF (NIC.LE.1) GOTO 800
      DO 830 I=1,NIC
      DO 820 J=1,NTOT
      IF (ID(J).EQ.NICNOS(I)) GOTO825
  820 CONTINUE
      WRITE (6,822) NICNOS(I)
  822 FORMAT (' NO STUDENT FOR ID #',I4)
      GOTO 800
  825 NICNOS(I)=J
  830 CONTINUE

  860 CONTINUE
      DO 840 J=1,NG
      KOUNT=0
      DO 835 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 835
      KOUNT=KOUNT+1
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
  835 CONTINUE
      IF (KOUNT.LE.1) GOTO 840

      X=0.0
      IX=0
      DO 850 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 850
      IF (SDIST(I).LT.X) GOTO 850
      X=SDIST(I)
      IX=I
  850 CONTINUE
      ASSIGN 860 TO IRETN
      GOTO 690
  840 CONTINUE
      GOTO 800


C ---------------------------------------------------------------------
C     OUTPUT FINAL TOTALS AND LEAVE
  900 CONTINUE
      DO 950 J=1,NG
      NSK=NGSKIL(J)
      WRITE (6,910) J,NSK,(SKNAME(JJ,NSK),JJ=1,3),NINGRP(J)
  910 FORMAT ('1','GROUP #',I2,', SKILL #',I2,2X,3A6,/,
     1 ' NUMBER OF STUDENTS RECOMMENDED = ',I3,//,
     2 ' STUD #',12X,'STUDENT NAME',10X,'DISTANCE',/)
      DO 940 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 930 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 930
      SUM=SUM+SLEARN(I,II)
      SUMSQ=SLEARN(I,II)**2+SUMSQ
      XN=XN+1.0
      IF (I.NE.1) GOTO 930
      SDIST(II)=DIST(SLEARN(1,II),SEED(1,J),NLER)
      WRITE (6,920) ID(II),(XNAME(JJ,II),JJ=1,4),SDIST(II)
  920 FORMAT (2X,I4,4X,4A6, 5X,F7.3)
  930 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
      IF (I.EQ.1) WRITE (6,70)
      WRITE (6,100) (LVNAME(JJ,I),JJ=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  940 CONTINUE
  980 CONTINUE
      CALL EXIT
      END
```

```
C---- SEED POINT ROUTINE - ALGORITHM C ----

      SUBROUTINE SEFDY (NSK)
      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=30
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     +       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT,WEIGHT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     +       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION NPART (MAXGRP)


C -----------------------------------------------------------------
C     FIND 2-STUDENTS W. GREATEST PAIR WISE DISTANCE
      NSTU1=1
      NSTU2=1
      DMAX=0.0
      DO 10 I=1,NTOT
      IF (ISKILL(NSK,I).EQ.0) GOTO 10
      NN=I+1
      DO 15 II=NN,NTOT
      IF (ISKILL (NSK,II).EQ.0) GOTO 15
      X=DIST (SLEARN(1,I),SLEARN(1,II),NLER)
      IF (X.LE.DMAX) GOTC 15
      NSTU1=I
      NSTU2=II
      DMAX=X
15    CONTINUE
10    CONTINUE
      WRITE (6,17) NSK,NSTU1,NSTU2,DMAX
   17 FORMAT('0SKILL =',I2,', 1ST STUDENT = ',I3,', 2ND STUDENT = ',
     1 I3,', DIFFERENCE = ',F9.3)


C -----------------------------------------------------------------
C     CONSTRUCT A TABLE OF STUDENT DISTANCES
      NELIG=0
      DO 100 I=1,NTOT
      IF (ISKILL(NSK,I).EQ.0) GOTO 100
      X=DIST (SLEARN (1,I), SLEARN(1,NSTU1),NLER)
      IF (NELIG.EQ.0) GOTO 30
      DO 20 II=1,NELIG
      IF (X.LT.SDIST(II)) GOTO 40
20    CONTINUE
30    SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=I
      GOTO 90
40    NSTU2=I
      DO 50 III=II,NELIG
      TEMP=SDIST(III)
      SDIST (III)=X
      X=TEMP
      NTEMP=NSTUNO(III)
      NSTUNO(III)=NSTU2
50    NSTU2=NTEMP
      SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=NSTU2
90    NELIG=NELIG+1
100   CONTINUE
      WRITE (6,80) (SDIST(I),NSTUNO(I),I=1,NELIG)
   80 FORMAT (5(2X,F9.3,2X,I3))


C --------------------------------------------------------------------
C     SUM THE LOWER GROUP LIMITS
      SUM=0.0
      DO 110 I=1,NG
      IF (NGSKIL(I).NE.NSK) GOTO 110
      SUM=SUM+FLOAT(NLOW(I))
110   CONTINUE


C --------------------------------------------------------------------
C     ESTABLISH A PARTITIONING
      NEQSK=0
      DO 120 I=1,NG
      IF (NGSKIL(I).NE.NSK) GOTO 120
      NEQSK=NEQSK+1
      PORTN=(FLOAT(NLOW(I))/SUM)*FLOAT(NELIG)
      IPORTN=PORTN
      IF((PORTN-IPORTN).GE.0.5) IPORTN=IPORTN+1
      NPART(NEQSK)=IPORTN
120   CONTINUE


C --------------------------------------------------------------------
C     BALANCE THE PARTITIONS
      WRITE (6,135) NSK
  135 FORMAT ('0 *** PARTITIONING FOR SKILL #',I2,' ***')
      WRITE (6,145) NELIG,(NPART(J),J=1,NEQSK)
145   FORMAT (' COMPUTED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      DO 140 J=1,NEQSK
      NSUM=0
      DO 130 I=1,NEQSK
130   NSUM=NSUM+NPART(I)
      IF(NSUM.EQ.NELIG) GOTO 150
      IF(NSUM.GT.NELIG) NPART(J)=NPART(J)-1
      IF(NSUM.LT.NELIG) NPART(J)=NPART(J)+1
140   CONTINUE
150   CONTINUE
      WRITE (6,155) NELIG,(NPART(J),J=1,NEQSK)
155   FORMAT (' BALANCED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      IF(NSUM.NE.NELIG) STOP
```

```
C ---------------------------------------------------------------------
C     SELECT MID-POINTS
      I=(NELIG+1)/2
      II=(NELIG+2)/2
      X1=SDIST(I)-SDIST(1)
      X2=SDIST(NELIG)-SDIST(II)


C ---------------------------------------------------------------------
C     FIND MEAN OF EACH PARTITION
      NEQSK=0
      II=0
      DO 200 I=1,NG
      IF (NGSKIL(I).NE.NSK) GOTO 200
      NEQSK=NEQSK+1
      NN=NPART(NEQSK)+II
      N=II+1
      N2=NN
      N1=N
      IF(X1.LT.X2) GOTO 160
      N2=NELIG+1-N
      N1=NELIG+1-NN
160   DO 190 K=1,NLER
      XN=0.
      SUM=0.
      DO 180 J=N1,N2
      JJ=NSTUNO(J)
      SUM=SUM+SLEARN(K,JJ)
180   XN=XN+1.
190   SEED(K,I)=SUM/XN
      II=NN
200   CONTINUE
      RETURN
      END
```

APPENDIX D

COMPUTER PROGRAM FOR GROUPAL D

4/12

```
C---- GROUPING ALGORITHM D -----

      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=30
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     *       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT,WEIGHT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     *       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION XNAME(4,MAXSTU),VLEARN(MAXLV,MAXSTU)
      DIMENSION ID(MAXSTU)
      DIMENSION XMEAN(MAXLV),XSTDEV(MAXLV),XVAR(MAXLV)
      DIMENSION NUM(MAXSK)
      DIMENSION NSKNOS(MAXSK),NLVNOS(MAXLV)
      INTEGER   SKNAME(3,MAXSK),LVNAME(3,MAXLV)
      DIMENSION NINGRP(MAXGRP)
      DIMENSION NICNOS(10)
      DIMENSION NGELIG (MAXSK,MAXGRP)
      LOGICAL INCOM,SIZEC


C -----------------------------------------------------------------------
C     INPUT AND ECHO PARAMETER CARDS
      WRITE(6,1)
    1 FORMAT('1','GROUPING ALGORITHM D',/,1X,20('-'),/)

      READ(5,2)NG,NSKILL,NLER,NTOT,WEIGHT,ILIMIT
    2 FORMAT(I2,1X,I2,1X,I2,1X,I3,1X,F4.2,1X,I2)
      WRITE(6,3)NG,NSKILL,NLER,NTOT,WEIGHT
    3 FORMAT('0NUMBER OF GROUPS REQUESTED = ',I2,/,
     1       ' NUMBER OF SKILLS CONSIDERED = ',I2,/,
     2       ' NUMBER OF LEARNER VARIABLES CONSIDERED = ',I2,/,
     3       ' NUMBER OF STUDENTS = ',I3,/,
     4       ' WEIGHT APPLIED TO SKILLS = ',F5.2,/)
      IF (ILIMIT.EQ.0) ILIMIT=10

      READ(5,4)K,L,M,N
    4 FORMAT(4I1)
      IF(K.EQ.1)WRITE(6,5)
    5 FORMAT(' ELIGIBILITY FOR SKILLS TAKEN INTO ACCOUNT')
      IF(K.EQ.0)WRITE(6,6)
    6 FORMAT(' ELIGIBILITY FOR SKILLS NOT TAKEN INTO ACCOUNT')
      IF(L.EQ.1)WRITE(6,7)
    7 FORMAT(' LEARNER VARIABLES TAKEN INTO ACCOUNT')
      IF(L.EQ.0)WRITE(6,8)
    8 FORMAT(' LEARNER VARIABLES NOT TAKEN INTO ACCOUNT')
      IF(M.EQ.1)WRITE(6,9)
    9 FORMAT(' MORE THAN ONE GROUP MAY STUDY THE SAME SKILL')
      IF(M.EQ.0)WRITE(6,11)
   11 FORMAT(' ONLY ONE GROUP MAY STUDY A PARTICULAR SKILL')
      IF(N.EQ.1)WRITE(6,21)
   21 FORMAT(' SEED POINTS FOR GROUPS COMPUTED')
      IF(N.EQ.0)WRITE(6,22)
   22 FORMAT(' SEED POINTS FOR GROUPS SPECIFIED')
```

```
      READ(5,10)(NLOW(I),NHIGH(I),I=1,NG)
   10 FORMAT (20I2)
      WRITE (6,19) (NLOW(I),NHIGH(I),I=1,NG)
   19 FORMAT('0GROUP RANGES = ',10(2I3,2X))


C ----------------------------------------------------------------------
C     RANGE TEST
      MSUM=0
      NSUM=0
      KLOW=99
      DO 20 I=1,NG
      NSUM=NSUM+NLOW(I)
      MSUM=MSUM+NHIGH(I)
      IF (NLOW(I).GT.NHIGH(I)) WRITE (6,16) I,NLOW(I),NHIGH(I)
   16 FORMAT (' INVALID RANGE  GRP #',I2,' LOW = ',I2,', HIGH = ',I2)
      IF (NLOW(I).GT.KLOW) WRITE (6,18) I,NLOW(I),KLOW
   18 FORMAT (' SEQUENCE ERROR  GPR #',I2,' CURRENT LOW = ',
     1 I2,', PREVIOUS LOW = ',I2)
      KLOW = NLOW(I)
   20 CONTINUE
      IF(NTOT.GT.MSUM.OR.NTOT.LT.NSUM)WRITE(6,30)NTOT
   30 FORMAT(' RANGE SIZES DO NOT CORRESPOND TO TOTAL NUMBER OF STUDEN
     1',I3)


C ----------------------------------------------------------------------
C     READ IN SKILL AND LEARNER VARIABLE NAMES
      READ (5,10) (NSKNOS(I),I=1,NSKILL)
      WRITE (6,185) (NSKNOS(I),I=1,NSKILL)
  185 FORMAT ('0SKILLS CONSIDERED = ',20I3)
      READ (5,10) (NLVNOS(I),I=1,NLER)
      WRITE (6,187) (NLVNOS(I),I=1,NLER)
  187 FORMAT ('0LEARNER VARIABLES CONSIDERED = ',20I3)
      DO 170 I=1,NSKILL
      READ(5,180)(SKNAME(J,I),J=1,3)
  180 FORMAT(3A6)
  170 CONTINUE
      DO 175 I=1,NLFR
      READ(5,180)(LVNAME(J,I),J=1,3)
  175 CONTINUE


C ----------------------------------------------------------------------
C     READ IN SEED POINTS
      IF (N.EQ.1) GOTO 190
      DO 195 I=1,NG
      READ (5,197) (SEED(J,I),J=1,NLFR)
  197 FORMAT (20F4.2)
  195 CONTINUE
```

```
 199 CONTINUE
     WRITE(6,14)
  14 FORMAT ('0',49X,'STUDENT RECORDS')
     WRITE(6,17)
  17 FORMAT('0', 8X,'STUDENT NAME',9X,' VL  VN  AC  AN  KT  SI  SO',
    1'  EO  EW   SO   SW  SEX',' 1 2 3 4 5 6 7 8 91011121314'/)
C ---------------------------------------------------------------------
C      READ IN STUDENT RECORDS
       DO 15 I=1,NTOT
       READ(5,12)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
  12 FORMAT(I4,1X,4A6,2X,9F2.0,F3.1,1X,F2.0,F1.0,1X,14I1)
       WRITE(6,13)ID(I),(XNAME(J,I),J=1,4),(VLEARN(J,I),J=1,12),
     1(ISKILL(J,I),J=1,14)
  13 FORMAT(I5,1X,4A6,9F4.0,F5.1,1X,F4.0,2X,F2.0,14I2)
  15 CONTINUE
C ---------------------------------------------------------------------
C      LEFT-JUSTIFY SELECTED SKILLS AND LEARNER VARIABLES
       DO 250 I=1,NTOT
       DO 220 II=1,NLER
       KK=NLVNOS(II)
       IF (L.EQ.0) VLEARN(KK,I)=0.0
 220 VLFARN(II,I)=VLEARN(KK,I)
       JJ=NLER
       DO 240 II=1,NSKILL
       KK=NSKNOS(II)
       IF (K.EQ.0) ISKILL(KK,I)=1
       ISKILL(II,I)=ISKILL(KK,I)
       JJ=JJ+1
       VLEARN(JJ,I)=FLOAT(ISKILL(KK,I))
       CO 230 KK=1,3
 230 LVNAME(KK,JJ)=SKNAME(KK,II)
 240 CONTINUE
 250 CONTINUE
       NLER=NLER+NSKILL
C ---------------------------------------------------------------------
C      CALCULATE NUMBER OF STUDENTS ELIGIBLE FOR EACH SKILL
       WRITE (6,130)
 130 FORMAT ('0NUMBER OF STUDENTS ELIGIBLE FOR THE FOLLOWING SKILLS -
       DO 140 J=1,NSKILL
       NUM(J)=0
       DO 150 I=1,NTOT
       NUM(J)=NUM(J)+ISKILL(J,I)
 150 CONTINUE
       WRITE(6,160)(SKNAME(II,J),II=1,3),NUM(J)
 160 FORMAT (1X,3A6,' -- ',I3)
```

```
  140 CONTINUE


C ----------------------------------------------------------------------
C       CALCULATE MEAN, VARIANCE, STD. DEV. OF EACH LEARNER VARIABLE
      WRITE (6,70)
   70 FORMAT ('0 *** LEARNER VARIABLES - MEAN, VAR. AND STD. DEV. ***'
      DO 90 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 80 J=1,NTOT
      IF(VLEARN(I,J).LT.0)GO TO 80
      SUM=SUM+VLEARN(I,J)
      SUMSQ=VLEARN(I,J)**2+SUMSQ
      XN=XN+1.
   80 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
   90 WRITE(6,100)(LVNAME(J,I),J=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  100 FORMAT(' ',3A6,' HAS MEAN = ',F6.2,2X,'VAR. = ',F8.2,2X,'SD. = '
     1,F6.2)


C ----------------------------------------------------------------------
C      MISSING DATA
      DO 200 I=1,NTOT
      DO 210 J=1,NLER
      IF(VLEARN(J,I).GE.0)GO TO 210
      VLEARN(J,I)=XMEAN(J)
  210 CONTINUE
  200 CONTINUE


C ----------------------------------------------------------------------
C      STANDARDIZE STUDENT VARIABLE SCORES
      DO 95 J=1,NTOT
      DO 97 I=1,NLER
      IF (XSTDEV(I).LT. 0.0001) GOTO 98
      SLEARN(I,J)=(VLEARN(I,J)-XMEAN(I))/XSTDEV(I)
      GOTO 97
   98 SLEARN(I,J)=XMEAN(I)
   97 CONTINUE
   95 CONTINUE

      WRITE (6,214)
  214 FORMAT ('0 *** STANDARDIZED LEARNER VARIABLES ***')
      DO 215 I=1,NTOT
  215 WRITE (6,216)  ID(I),(XNAME(J,I),J=1,4),(SLEARN(J,I),J=1,NLER)
  216 FORMAT (I5,1X,4A6,14(F6.2))
```

```


C ------------------------------------------------------------------
C     PARTITION GROUPS BASED ON SHORTEST DISTANCE
      IF (N.EQ.1) CALL SEEDY


C ------------------------------------------------------------------
C     OUTPUT GROUP MEANS
      PDIST=0.0
      KKK=0
  450 CONTINUE
      WRITE (6,410)
  410 FORMAT ('0 *** MEAN OF EACH GROUP ***')
      DO 420 I=1,NG
  420 WRITE (6,430) I,(SEED(J,I),J=1,NLER)
  430 FORMAT(' GROUP #',I2,13(F7.3))


C ------------------------------------------------------------------
C     ASSIGN STUDENTS TO SEED POINTS
      TDIST=0.0
      TSSW=0.0
      WRITE (6,470)
  470  FORMAT ('0 *** STUDENT GROUP ASSIGNMENTS ***')
      DO 500 I=1,NTOT
      NSTUNO(I)=0
      DO 490 J=1,NG
      X=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (NSTUNO(I).EQ.0) GOTO 480
      IF (X.GT.XDIST) GOTO 490
  480 XDIST=X
      NSTUNO(I)=J
  490 CONTINUE
      IF (NSTUNO(I).EQ.0) XDIST=0.0
      SDIST(I)=XDIST
      J=NSTUNO(I)
      IF (J.NE.0) XDIST = DIST (SLEARN(1,I),SEED(1,J),NLER-NSKILL)
      TDIST=TDIST+XDIST
      TSSW=TSSW+XDIST**2
  500 CONTINUE
      WRITE (6,495) (I,NSTUNO(I),SDIST(I),I=1,NTOT)
  495 FORMAT (3(7X,'STU #',I3,' ASG TO ',I2,' W. DIST ',F5.2))


C ------------------------------------------------------------------
C     COMPUTE MEAN AND VAR. OF ALL GROUPS
      DO 600 J=1,NG
      DO 590 I=1,NLER
      SUM=0.0
      XN=0.0
      DO 580 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 580
      SUM=SUM+SLEARN(I,II)
      XN=XN+1.
  580 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      SEED(I,J)=SUM/XN
  590 CONTINUE
  600 CONTINUE

      IF (KKK.EQ.0) V=TDIST
      IF (KKK.NE.0) V=ABS(TDIST-PDIST)
      KKK=KKK+1
      PDIST=TDIST
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,620) KKK,TDIST,V,TSSW,XMSSW
  620 FORMAT ('0FOR ITERATION #',I2,' TOTAL DISTANCE = ',F7.3,
     1' WHICH DIFFERS FROM PREVIOUS BY ',F7.3,
     2 /,5X,'TOTAL SUM OF SQUARES WITHIN = ',F8.2,
     3   ',  MEAN SUM OF SQUARES WITHIN = ',F7.2)
      IF (V.GT.0.001 .AND. KKK.LE.ILIMIT) GOTO 450


C ----------------------------------------------------------------------
C     OUTPUT NUMBER ELIGIBLE IN EACH GROUP
      WRITE (6,305) (I,I=1,NSKILL)
  305 FORMAT ('0GRP = ',15I4)
      DO 330 J=1,NG
      DO 320 I=1,NSKILL
      NGELIG(I,J)=0
      DO 310 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 310
      IF (ISKILL(I,II).EQ.0) GOTO 310
      NGELIG(I,J)=NGELIG(I,J)+1
  310 CONTINUE
  320 CONTINUE
      WRITE (6,325) J,(NGELIG(I,J),I=1,NSKILL)
  325 FORMAT (2X,I2,3X,15I4)
  330 CONTINUE


C ----------------------------------------------------------------------
C     ASSIGN SKILLS TO GROUPS
      DO 335 I=1,NG
  335 NGSKIL(I)=0
      KKK=0
  340 CONTINUE
      MAX=0
      DO 370 J=1,NG
      IF (NGSKIL(J).NE.0) GOTO 370
      DO 360 I=1,NSKILL
      IF (M.EQ.1) GOTO 355
      DO 350 II=1,NG
      IF (NGSKIL(II).EQ.I) GOTO 360
  350 CONTINUE
  355 IF (NGELIG(I,J).LE.MAX) GOTO 360
      MAX=NGELIG(I,J)
      IG=J
      IS=I
  360 CONTINUE
  370 CONTINUE
      NGSKIL(IG)=IS
      KKK=KKK+1
      IF (KKK.LT.NG) GOTO 340


C -----------------------------------------------------------------
C     OUTPUT OMISSIONS AND NO. IN EACH GROUP
      WRITE (6,625)
  625 FORMAT ('1',' *** STUDENT OMISSIONS ***',/)
      DO 630 J=1,NG
  630 NINGRP(J)=0
      DO 640 I=1,NTOT
      JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 635
      NINGRP(JJ)=NINGRP(JJ)+1
      GOTO 640
  635 WRITE (6,13) ID(I),(XNAME(II,I),II=1,4)
  640 CONTINUE
      WRITE (6,655)
  655 FORMAT('0GRP #   SKILL   NO. IN GROUP')
      WRITE (6,660) (J,NGSKIL(J),NINGRP(J),J=1,NG)
  660 FORMAT (I4,4X,I4,8X,I4)


C -----------------------------------------------------------------
C     CHECK FOR STUDENTS NOT ELIGILRE FOR SKILL OF GROUP
      WRITE (6,963)
  963 FORMAT ('0REMOVE INELIGIBLE STUDENTS FROM GROUPS',/)
      ASSIGN 960 TO IRETN
      INCOM=.FALSE.
      SIZEC=.FALSE.
  960 CONTINUE
      IX=0
      X=0.0
      DO 970 I=1,NTOT
      JJ=NSTUNO(I)
      IF (JJ.EQ.0) GOTO 970
      II=NGSKIL(JJ)
      IF (ISKILL(II,I).EQ.1) GOTO 970
      SDIST(I)=DIST(SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 970
      X=SDIST(I)
      IX=I
  970 CONTINUE
```

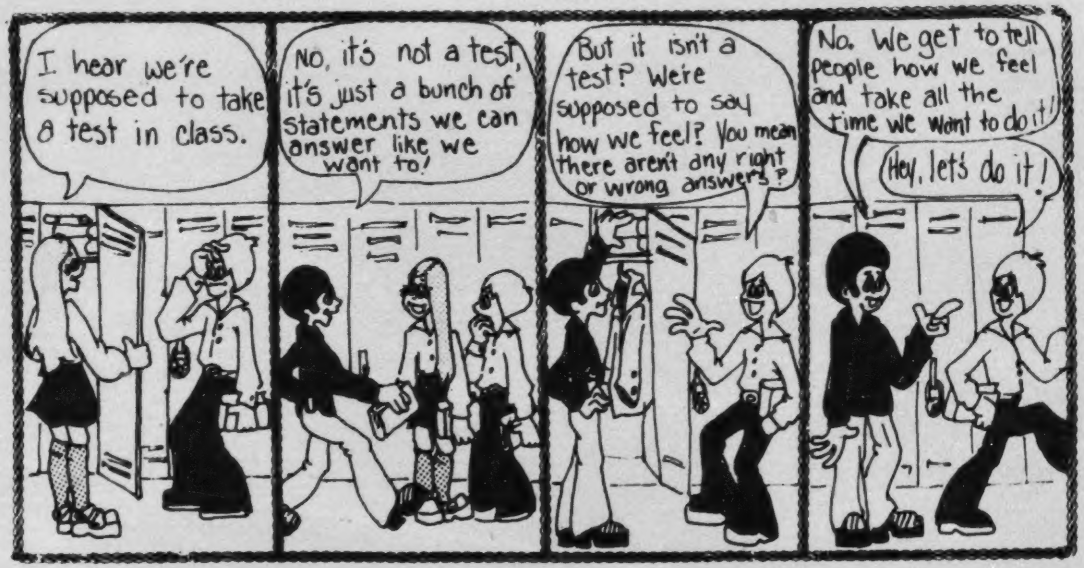
```
      IF (IX.NE.0) GOTO 690
      SIZEC=.TRUE.
      WRITE (6,980)
  980 FORMAT ('0PROCESS SIZE CONSTRAINTS',/).

C ---------------------------------------------------------------------
C     CHECK FOR GROUPS OVERLOADED
      ASSIGN 670 TO IRETN
      INCOM=.FALSE.
  670 CONTINUE
      DO 675 J=1,NG
      IF (NINGRP(J).GT.NHIGH(J)) GOTO 677
  675 CONTINUE
      GOTO 800

C ---------------------------------------------------------------------
C     COMPUTE DISTANCES AND FIND FARTHEST STUDENT IN OVERLOADED GROUP
  677 CONTINUE
      X=0.0
      IX=0
      DO 685 J=1,NG
      IF (NINGRP(J).LE.NHIGH(J)) GOTO 685
      DO 680 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 680
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
      IF (SDIST(I).LT.X) GOTO 680
      X=SDIST(I)
      IX=I
  680 CONTINUE
  685 CONTINUE

C ---------------------------------------------------------------------
C     FIND ANOTHER GROUP FOR THIS STUDENT
  690 CONTINUE
      Y=0.0
      IY=0
      J=NSTUNO(IX)
      DO 700 JJ=1,NG
      IF (J.EQ.JJ) GOTO 700
      IF (SIZEC .AND. NINGRP(JJ).GE.NHIGH(JJ)) GOTO 700
      NSK=NGSKIL(JJ)
      IF (ISKILL(NSK,IX).EQ.0) GOTO 700

C ---------- CHECK FOR INCOMPATIBLES IN SAME GROUP
      IF (.NOT.INCOM) GOTO 698
      DO 695 II=1,NIC
      I=NICNOS(II)
      IF (I.EQ.IX) GOTO 695
      IF (NSTUNO(I).EQ.JJ) GOTO 700
  695 CONTINUE
  698 CONTINUE
      YY=DIST (SLEARN(1,IX),SEED(1,JJ),NLER)
      IF (IY.NE.0 .AND. YY.GT.Y) GOTO 700
      IY=JJ
      Y=YY
  700 CONTINUE
      IF (IY.NE.0) GOTO 750


C -----------------------------------------------------------------
C     CANNOT MOVE - SO PUT IN OMISSIONS
      WRITE (6,720) IX,J
  720 FORMAT ('0 STUDENT #',I3,' HAS BEEN BOOTED OUT OF GRP #',I2)
      WRITE (6,710) ID(IX),(XNAME(II,IX),II=1,4),J
  710 FORMAT (I5,1X,4A6,' HAS BEEN REMOVED FROM GRP #',I2)
      NSTUNO(IX)=0
      SDIST(IX)=0.0
      NINGRP(J)=NINGRP(J)-1
      GOTO 760


C -----------------------------------------------------------------
C     MOVE STUDENT AND RECOMPUTE GROUP MEANS
  750 CONTINUE
      WRITE (6,755) IX,J,IY
  755 FORMAT ('0 STUDENT #',I3,' IS TO BE MOVED FROM GRP #',
     1  I2,' TO GRP #',I2)
      NSTUNO(IX)=IY
      NINGRP(J)=NINGRP(J)-1
      NINGRP(IY)=NINGRP(IY)+1

      DO 790 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 780 I=1,NTOT
      IF (NSTUNO(I).NE.IY) GOTO 780
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  780 CONTINUE
      SEED(JJ,IY)=SUM/XN
  790 CONTINUE

  760 CONTINUE
      DO 775 JJ=1,NLER
      SUM=0.0
      XN=0.0
      DO 770 I=1,NTOT
      IF (NSTUNO(I).NE.J) GOTO 770
      SUM=SUM+SLEARN(JJ,I)
      XN=XN+1.0
  770 CONTINUE
      SEED(JJ,J)=SUM/XN
  775 CONTINUE


C -----------------------------------------------------------------
C     RECOMPUTE STUDENT DISTANCES
      TDIST=0.0
      TSSW=0.0
      DO 740 I=1,NTOT
      SDIST(I)=0.0
      J=NSTUNO(I)
      IF (J.EQ.0) GOTO 740
      SDIST(I)=DIST (SLEAPN(1,I),SEED(1,J),NLER)
      X = DIST (SLEARN(1,I),SEED(1,J),NLER-NSKILL)
      TDIST=TDIST+X
      TSSW=TSSW+X**2
      SDIST(I)=X
  740 CONTINUE
      XMSSW=TSSW/FLOAT(NTOT-NG)
      WRITE (6,745) TDIST,TSSW,XMSSW
  745 FORMAT(5X,'TOTAL DIST. = ',F7.3,' TSSW = ',F8.2,' MSSW = ',F7.2)
      GOTO IRETN


C -----------------------------------------------------------------
C     PROCESS INCOMPATIBLE STUDENTS IN SIMILAR MANNER
  800 CONTINUE
      INCOM=.TRUE.
      READ (5,810,END=900) NIC,NICNOS
  810 FORMAT (I2,10I4)
      WRITE (6,815) (NICNOS(I),I=1,NIC)
  815 FORMAT ('0INCOMPATIBLE STUDENTS = ',10I5)
      IF (NIC.LE.1) GOTO 800
      DO 830 I=1,NIC
      DO 820 J=1,NTOT
      IF (ID(J).EQ.NICNOS(I)) GOTO825
  820 CONTINUE
      WRITE (6,822) NICNOS(I)
  822 FORMAT (' NO STUDENT FOR ID #',I4)
      GOTO 800
  825 NICNOS(I)=J
  830 CONTINUE

  860 CONTINUE
      DO 840 J=1,NG
      KOUNT=0
      DO 835 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 835
      KOUNT=KOUNT+1
      SDIST(I)=DIST (SLEARN(1,I),SEED(1,J),NLER)
  835 CONTINUE
      IF (KOUNT.LE.1) GOTO 840
      X=0.0
      IX=0
      DO 850 II=1,NIC
      I=NICNOS(II)
      IF (NSTUNO(I).NE.J) GOTO 850
      IF (SDIST(I).LT.X) GOTO 850
      X=SDIST(I)
      IX=I
  850 CONTINUE
      ASSIGN 860 TO IRETN
      GOTO 690
  840 CONTINUE
      GOTO 800


C ---------------------------------------------------------------------
C     OUTPUT FINAL TOTALS AND LEAVE
  900 CONTINUE
      DO 950 J=1,NG
      NSK=NGSKIL(J)
      WRITE (6,910) J,NSK,(SKNAME(JJ,NSK),JJ=1,3),NINGRP(J)
  910 FORMAT ('1','GROUP #',I2,', SKILL #',I2,2X,3A6,/,
     1 ' NUMBER OF STUDENTS RECOMMENDED = ',I3,//,
     2 ' STUD #',12X,'STUDENT NAME',10X,'DISTANCE',/)
      DO 940 I=1,NLER
      SUMSQ=0.0
      SUM=0.0
      XN=0.0
      DO 930 II=1,NTOT
      IF (NSTUNO(II).NE.J) GOTO 930
      SUM=SUM+SLEARN(I,II)
      SUMSQ=SLEARN(I,II)**2+SUMSQ
      XN=XN+1.0
      IF (I.NE.1) GOTO 930
      SDIST(II)=DIST(SLEARN(1,II),SEED(1,J),NLER)
      WRITE (6,920) ID(II),(XNAME(JJ,II),JJ=1,4),SDIST(II)
  920 FORMAT (2X,I4,4X,4A6, 5X,F7.3)
  930 CONTINUE
      IF (XN .LT. 0.5) XN=1.0
      XMEAN(I)=SUM/XN
      XVAR(I)=ABS(XN*SUMSQ-SUM**2)/(XN**2)
      XSTDEV(I)=SQRT(XVAR(I))
      IF (I.EQ.1) WRITE (6,70)
      WRITE (6,100) (LVNAME(JJ,I),JJ=1,3),XMEAN(I),XVAR(I),XSTDEV(I)
  940 CONTINUE
  950 CONTINUE
      CALL EXIT
      END
```

```
C---- SEED POINT ROUTINE - ALGORITHM D ----

      SUBROUTINE SEEDY
      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=30
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     +       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT,WEIGHT
      COMMON /SEED/ NGSKIL (MAXGRP), SEED(MAXLV,MAXGRP),
     +       SDIST (MAXSTU), NSTUNO (MAXSTU)
      DIMENSION NPART (MAXGRP)


C ----------------------------------------------------------------
C     FIND 2-STUDENTS W. CREATEST PAIR WISE DISTANCE
      NSTU1=1
      NSTU2=1
      DMAX=0.0
      DO 10 I=1,NTOT
      NN=I+1
      DO 15 II=NN,NTOT
      X=DIST (SLEARN(1,I),SLEARN(1,II),NLER)
      IF (X.LE.DMAX) GOTO 15
      NSTU1=I
      NSTU2=II
      DMAX=X
15    CONTINUE
10    CONTINUE
      WRITE (6,17) NSTU1,NSTU2,DMAX
   17 FORMAT('0LARGEST DIST:  1ST STUDENT = ',I3,', 2ND STUDENT = ',
     1 I3,', DIFFERENCE = ',F9.3)


C ----------------------------------------------------------------
C     CONSTRUCT A TABLE OF STUDENT DISTANCES
      NELIG=0
      DO 100 I=1,NTOT
      X=DIST (SLEARN (1,I), SLEARN(1,NSTU1),NLER)
      IF (NELIG.EQ.0) GOTO 30
      DO 20 II=1,NELIG
      IF (X.LT.SDIST(II)) GOTO 40
20    CONTINUE
30    SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=I
      GOTO 90
40    NSTU2=I
      DO 50 III=II,NELIG
      TEMP=SDIST(III)
      SDIST (III)=X
      X=TEMP
      NTEMP=NSTUNO(III)
      NSTUNO(III)=NSTU2
50    NSTU2=NTEMP
      SDIST(NELIG+1)=X
      NSTUNO(NELIG+1)=NSTU2
90    NELIG=NELIG+1
100   CONTINUE
      WRITE (6,80) (SDIST(I),NSTUNO(I),I=1,NELIG)
   80 FORMAT (5(2X,F9.3,2X,I3))


C ---------------------------------------------------------------------
C     SUM THE LOWER GROUP LIMITS
      SUM=0.0
      DO 110 I=1,NG
      SUM=SUM+FLOAT(NLOW(I))
110   CONTINUE


C ---------------------------------------------------------------------
C     ESTABLISH A PARTITIONING
      NEQSK=0
      DO 120 I=1,NG
      NEQSK=NEQSK+1
      PORTN=(FLOAT(NLOW(I))/SUM)*FLOAT(NELIG)
      IPORTN=PORTN
      IF((PORTN-IPORTN).GE.0.5) IPORTN=IPORTN+1
      NPART(NEQSK)=IPORTN
120   CONTINUE


C ---------------------------------------------------------------------
C     BALANCE THE PARTITIONS
      WRITE (6,135)
  135 FORMAT ('0 *** GROUP PARTITIONING ***')
      WRITE (6,145) NELIG,(NPART(J),J=1,NEQSK)
145   FORMAT (' COMPUTED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      DO 140 J=1,NEQSK
      NSUM=0
      DO 130 I=1,NEQSK
130   NSUM=NSUM+NPART(I)
      IF(NSUM.EQ.NELIG) GOTO 150
      IF(NSUM.GT.NELIG) NPART(J)=NPART(J)-1
      IF(NSUM.LT.NELIG) NPART(J)=NPART(J)+1
140   CONTINUE
150   CONTINUE
      WRITE (6,155) NELIG,(NPART(J),J=1,NEQSK)
155   FORMAT (' BALANCED PARTITION WITH ',I3,' STUDENTS: ',10(I3,2X))
      IF(NSUM.NE.NELIG) STOP


C ---------------------------------------------------------------------
C     SELECT MID-POINTS
      I=(NELIG+1)/2
      II=(NELIG+2)/2
      X1=SDIST(I)-SDIST(1)
      X2=SDIST(NELIG)-SDIST(II)


C ---------------------------------------------------------------------
C     FIND MEAN OF EACH PARTITION
      NEQSK=0
      II=0
      DO 200 I=1,NG
      NEQSK=NEQSK+1
      NN=NPART(NEQSK)+II
      N=II+1
      N2=NN
      N1=N
      IF(X1.LT.X2) GOTO 160
      N2=NELIG+1-N
      N1=NELIG+1-NN
160   DO 190 K=1,NLER
      XN=0.
      SUM=0.
      DO 180 J=N1,N2
      JJ=NSTUNO(J)
      SUM=SUM+SLEARN(K,JJ)
180   XN=XN+1.
190   SEED(K,I)=SUM/XN
      II=NN
200   CONTINUE
      RETURN
      END
```

```
C---- DISTANCE FUNCTION (UNWEIGHTED) ----

      FUNCTION DIST(POINT1,POINT2,KOORD)
      DIMENSION POINT1(2),POINT2(2)
      SDIFSQ=0.0
      DO 220 I=1,KOORD
      DIF=(POINT1(I)-POINT2(I))
      DIFSQ=DIF**2
  220 SDIFSQ=SDIFSQ+DIFSQ
      DIST=SORT(SDIFSQ)
      RETURN
      END

C---- DISTANCE FUNCTION (WEIGHTED) ----

      FUNCTION DIST(POINT1,POINT2,KOORD)
      PARAMETER MAXGRP=15,MAXSTU=120,MAXSK=15,MAXLV=30
      COMMON SLEARN (MAXLV, MAXSTU), ISKILL (MAXSK,MAXSTU),
     +       NLOW(MAXGRP), NHIGH(MAXGRP), NG,NSKILL,NLER,NTOT,WEIGHT
      DIMENSION POINT1(2),POINT2(2)
      SDIFSQ=0.0
      DO 220 I=1,KOORD
      DIF=(POINT1(I)-POINT2(I))
      IF (I.GT.(NLER-NSKILL)) DIF=DIF*WEIGHT
      DIFSQ=DIF**2
  220 SDIFSQ=SDIFSQ+DIFSQ
      DIST=SORT(SDIFSQ)
      RETURN
      END
```

APPENDIX E

CITE LEARNING STYLES INVENTORY


I hear we're supposed to take a test in class.
No, it's not a test, it's just a bunch of statements we can answer like we want to!
But it isn't a test? We're supposed to say how we feel? You mean there aren't any right or wrong answers?
No. We get to tell people how we feel and take all the time we want to do it!
Hey, let's do it!

LEARNING STYLES SURVEY

INSTRUCTIONS: Read the statement carefully and mark out the number in parenthesis that best agrees with how you feel about the statement.

SAMPLE STATEMENT: 1. I would rather do schoolwork in the afternoon than in the morning.

On your answer sheet there are four (4) possible responses ranging from "most like me" to "least like me". Select the response that best describes the way you feel about the statement and mark out the number in the parenthesis.

	MOST LIKE ME			LEAST LIKE ME
1.	(4)	(3)	(2)	(1)

A number "1" response would mean you very much prefer to do schoolwork in the morning. There is no right or wrong response; only the way you feel about the statement. You may have all the time you want so please respond to every statement. Now, if there are no questions, go to your answer sheet and we will begin.

LEARNING STYLES

FORM C

1. When I make things for my studies I remember what I have learned better.
2. Most of the things I know I can write about better than I can tell about.
3. When I really want to understand what I have read, I read it softly to myself.
4. I get more done when I work alone.
5. I remember what I have read better than what I've heard.
6. I would rather tell a story than write it.
7. When I do my problems in my head, I say the numbers to myself.
8. I enjoy joining in at class meetings.
9. I understand a math problem that is written down better than one I hear.
10. I do better when I can write the answer instead of having to say it.
11. I understand spoken directions better than written ones.
12. I like to work by myself.
13. I would rather read a story, than listen to it read.
14. It's easy for me to tell about the things I know.
15. Saying the multiplications tables over and over helped me remember them better than writing then over and over.

Did you respond once to each statement?

Go to the next page

16. I prefer to work with a group when there is work to be done.

17. I understand a math problem that is written down better than one I hear.

18. Writing a spelling word several times helps me remember it better.

19. I remember things I hear better than things I read.

20. I learn best when I study alone.

21. I would rather read things in a book than have the teacher tell me about them.

22. I feel like I talk smarter than I write.

23. When I'm told the pages of my homework, I can remember them without writing them down.

24. If I have to decide something, I ask other people for their opinions.

25. Written math problems are easier for me to do than oral ones.

26. Subjects which call for doing projects or displays are easy to learn about.

27. I don't mind doing written assignments.

28. I study best when no one is around to talk or listen to.

29. I do well in classes where most of the information has to be read.

30. If homework were oral, I would do it all.

Did you respond once to each statement?

Go to the next page

31. Oral math tests with oral answers are easier for me than written tests.

32. I can learn more about a subject if I am with a small group of students.

33. Seeing a number makes more sense to me than hearing a number.

34. I like to make things with my hands.

35. I like tests that call for sentence completion or written answers.

36. I understand more from a class discussion than from reading about a subject.

37. I learn better by reading than by listening.

38. Speaking is a better way than writing if you want someone to understand what you really mean.

39. Numbers I hear make more sense to me than numbers I see.

40. I like to study with other people.

41. Seeing the price of something written down is easier for me to understand than having someone tell me the price.

42. I understand what I have learned better when I am involved in making something for the subject.

43. Sometimes I say dumb things, but writing gives me time to correct myself.

44. I do well on tests if they are about things I heard in class.

45. I can't think as well when I work with someone else as when I work alone.

Did you respond once to each statement?

Hey, that's all, man!

NAME ______________________ ______________________
(FIRST) (LAST)

	MOST LIKE ME		LEAST LIKE ME			MOST LIKE ME		LEAST LIKE ME	
1.	(1)	(2)	(3)	(4)	24.	(1)	(2)	(3)	(4)
2.	(1)	(2)	(3)	(4)	25.	(1)	(2)	(3)	(4)
3.	(1)	(2)	(3)	(4)	26.	(1)	(2)	(3)	(4)
4.	(1)	(2)	(3)	(4)	27.	(1)	(2)	(3)	(4)
5.	(1)	(2)	(3)	(4)	28.	(1)	(2)	(3)	(4)
6.	(1)	(2)	(3)	(4)	29.	(1)	(2)	(3)	(4)
7.	(1)	(2)	(3)	(4)	30.	(1)	(2)	(3)	(4)
8.	(1)	(2)	(3)	(4)	31.	(1)	(2)	(3)	(4)
9.	(1)	(2)	(3)	(4)	32.	(1)	(2)	(3)	(4)
10.	(1)	(2)	(3)	(4)	33.	(1)	(2)	(3)	(4)
11.	(1)	(2)	(3)	(4)	34.	(1)	(2)	(3)	(4)
12.	(1)	(2)	(3)	(4)	35.	(1)	(2)	(3)	(4)
13.	(1)	(2)	(3)	(4)	36.	(1)	(2)	(3)	(4)
14.	(1)	(2)	(3)	(4)	37.	(1)	(2)	(3)	(4)
15.	(1)	(2)	(3)	(4)	38.	(1)	(2)	(3)	(4)
16.	(1)	(2)	(3)	(4)	39.	(1)	(2)	(3)	(4)
17.	(1)	(2)	(3)	(4)	40.	(1)	(2)	(3)	(4)
18.	(1)	(2)	(3)	(4)	41.	(1)	(2)	(3)	(4)
19.	(1)	(2)	(3)	(4)	42.	(1)	(2)	(3)	(4)
20.	(1)	(2)	(3)	(4)	43.	(1)	(2)	(3)	(4)
21.	(1)	(2)	(3)	(4)	44.	(1)	(2)	(3)	(4)
22.	(1)	(2)	(3)	(4)	45.	(1)	(2)	(3)	(4)
23.	(1)	(2)	(3)	(4)					

PROJECT CITE

LEARNING STYLES WORKSHEET

NAME ____________________________

SCHOOL __________________________

DATE ____________________________

TEACHER _________________________

VISUAL LANGUAGE

5 -
13 -
21 -
29 -
37 -

Total ______ **x 2 =** ______ (Score)

VISUAL NUMERICAL

9 -
17 -
25 -
33 -
41 -

Total ______ **x 2 =** ______ **(Score)**

AUDITORY LANGUAGE

3 -
11 -
19 -
36 -
44 -

Total ______ **x 2 =** ______ **(Score)**

AUDITORY NUMERICAL

7 -
15 -
23 -
31 -
39 -

Total ______ **x 2 =** ______ **(Score)**

KINESTHETIC-TACTILE

1 -
18 -
26 -
34 -
42 -

Total ______ **x 2 -** ______ **(Score)**

SOCIAL-INDIVIDUAL

4 -
12 -
20 -
28 -
45 -

Total ______ **x 2 =** ______ (Score)

SOCIAL - GROUP

8 -
16 -
24 -
32 -
40 -

Total ______ **x 2** ______ **(Score)**

EXPRESSIVENESS-ORAL

6 -
14 -
22 -
30 -
38 -

Total ______ **x 2** ______ (Score)

EXPRESSIVENESS-WRITTEN

2 -
10 -
27 -
35 -
43 -

Total ______ **x 2** ______ (Score)

APPENDIX F

WIS-SIM REPORTS

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              WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

UNIT PERFORMANCE PROFILE                                              PAGE   1
UNIT A  RND DEMONSTRATION ELEMENTARY SCHOOL                     AS OF 07-29-76

                   ----------------- WORD ATTACK SKILLS -----------------
                   ---------------------- LEVEL C ----------------------
STUDENT NUMBER/NAME  1  2  3  4  5  6  7  8  9  10 11 12 13 14 15 16 17 18 ALL

  0060
ADAMS, ALAN          M  M
  0070
ANDERSON, ANDY          M
  0090
BAILEY, BRIAN        M  TC
  0100
BECK, BARBARA           M
  0150
BERG, BECKY          M  M
  0160
BOND, BARRY             TC
  0170
BRIGGS, BRUCE        M  M
  0200
BRUNER, BONNIE          12
  0205
CARLSON, CARL        M  29
  0210
CHRISTENSEN, CHRIS
  0230
COHEN, CATHY         M
  0260
CROSBY, CRAIG
  0310
DAVIS, DAVID         M
  0320
DISCH, DOROTHY
  0330
DOYLE, DIANE         M
  0350
ELLIOTT, ELMA
  0360
FARMER, FRED         M
  0370
FLEMING, FRANCIS
  0380
FREY, FRANK          M
  0390
GATES, GEORGE
  0400
GOLDSTEIN, GINA      M  TC
  0420
GREEN, GARY
  0430
HALL, HARRY          M
  0450
HARRIS, HELEN
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                  WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

UNIT PERFORMANCE PROFILE                                                  PAGE  2
UNIT A  RND DEMONSTRATION ELEMENTARY SCHOOL                         AS OF 07-29-76

                        ------------------ WORD ATTACK SKILLS -------------------
                        ----------------------- LEVEL C ------------------------
STUDENT NUMBER/NAME     1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 ALL

  0470
HENDERSON, HARVEY       M
  0480
HILL, HOWARD
  0510
HUBBARD, HOPE           M
  0540
JACKSON, JOHN
  0560
JOHNSON, JERRY          M
  0595
KAISER, KARL
  0610
KING, KATHY             M
  0620
KOCH, KENNETH
  0630
KRUEGER, KEVIN          M
  0640
LARSON, LYNN
  0650
LEWIS, LINDA            M
  0660
LORENZ, LYDIA
  0670
MALONEY, MARY           M
  0680
MARTIN, MARTHA
  0690
MCGUIRE, MIKE           M
  0700
MEYER, MARK
  0710
MOORE, MICHAEL          M
  0720
NELSON, NANCY
  0730
OLSEN, OTTO             M
  0750
PERRY, PAMELA
  0760
PIERCE, PAUL            M
  0770
PUTNAM, PATTY
  0780
REED, ROBERT            M
  0790
ROBERTS, RICHARD
```

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                   WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

UNIT PERFORMANCE PROFILE                                                PAGE  3
UNIT A  RND DEMONSTRATION ELEMENTARY SCHOOL                       AS OF 07-29-76

                    ------------------- WORD ATTACK SKILLS -------------------
                    ------------------------- LEVEL C -------------------------
STUDENT NUMBER/NAME  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 ALL

  0795
ROTH, RITA           M
  0805
SCHAFFER, SAMUEL
  0810
SCHMIDT, SUSAN       M
  0830
SCOTT, STEVEN
  0840
SIMON, SANDRA        M
  0850
SMITH, SHARON
  0860
STONE, SALLY         M
  0870
TAYLOR, TIMOTHY
  0880
TYLER, THEODORE      M
  0890
WAGNER, WALTER
  0900
WEBSTER, WENDY       TC TC TC
```

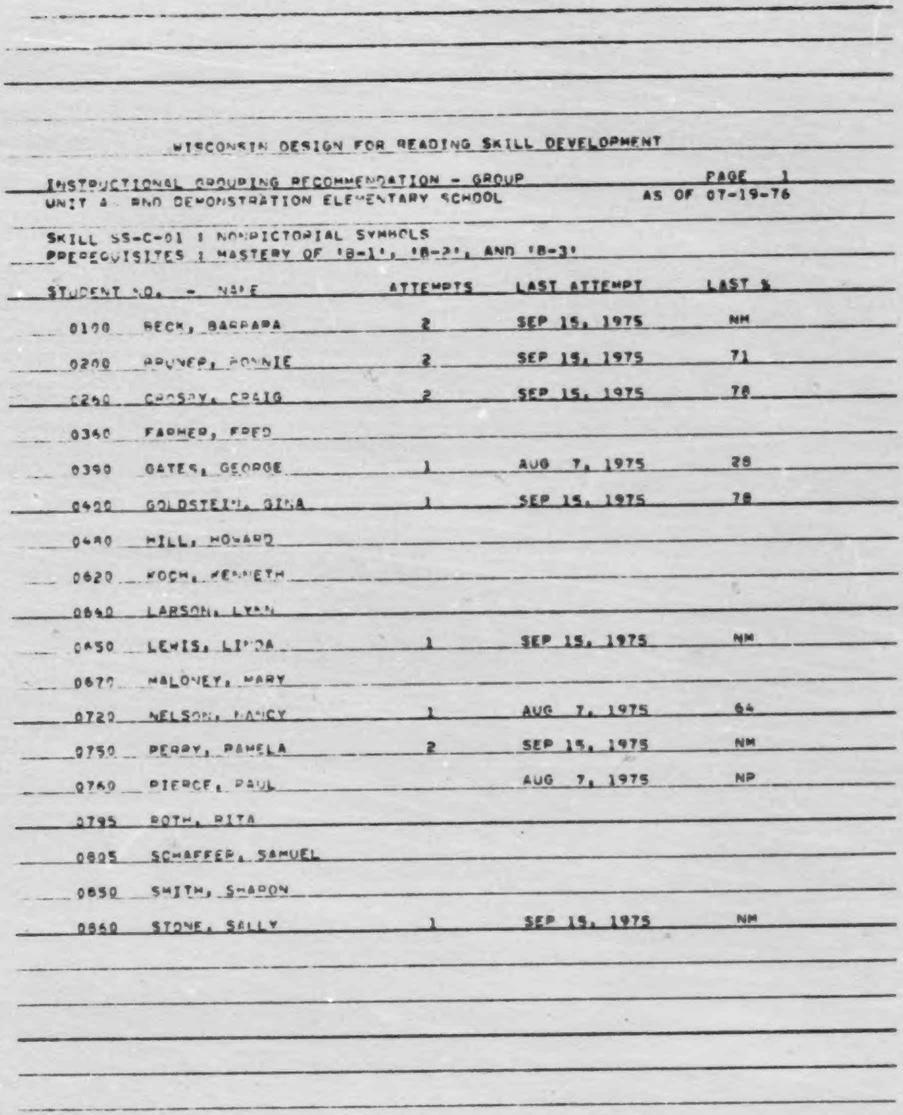

WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

INSTRUCTIONAL GROUPING RECOMMENDATION - GROUP — PAGE 1
UNIT A . RND DEMONSTRATION ELEMENTARY SCHOOL — AS OF 07-19-76

SKILL SS-C-01 : NONPICTORIAL SYMBOLS
PREREQUISITES : MASTERY OF 'B-1', 'B-2', AND 'B-3'

STUDENT NO.	- NAME	ATTEMPTS	LAST ATTEMPT	LAST %
0100	BECK, BARBARA	2	SEP 15, 1975	NM
0200	BRUNER, BONNIE	2	SEP 15, 1975	71
0260	CROSBY, CRAIG	2	SEP 15, 1975	78
0360	FARMER, FRED			
0390	GATES, GEORGE	1	AUG 7, 1975	28
0420	GOLDSTEIN, GINA	1	SEP 15, 1975	78
0480	HILL, HOWARD			
0620	KOCH, KENNETH			
0640	LARSON, LYNN			
0650	LEWIS, LINDA	1	SEP 15, 1975	NM
0670	MALONEY, MARY			
0720	NELSON, NANCY	1	AUG 7, 1975	64
0750	PERRY, PAMELA	2	SEP 15, 1975	NM
0760	PIERCE, PAUL		AUG 7, 1975	NP
0795	ROTH, RITA			
0805	SCHAFFER, SAMUEL			
0850	SMITH, SHARON			
0860	STONE, SALLY	1	SEP 15, 1975	NM

WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

INSTRUCTIONAL GROUPING RECOMMENDATION - SUMMARY PAGE 2
UNIT A R AND D DEMONSTRATION ELEMENTARY SCHOOL AS OF 07-19-76

STUDENT NO. - NAME	S B O 1	S B O 2	S C O 1	S C O 2	S D C 1	S D O 2	
0480 HILL, HOWARD			X	X			2
0595 KAISER, KARL				X			1
0610 KING, KATHY				X			1
0620 KOCH, KENNETH			X				1
0630 KRUEGER, KEVIN				X			1
0640 LARSON, LYNN			X	X			2
0650 LEWIS, LINDA			X				1
0660 LORENZ, LYDIA				X			1
0670 MALONEY, MARY			X				1
0680 MARTIN, MARTHA				X			1
0690 MCGUIRE, MIKE				X			1
0710 MOORE, MICHAEL				X			1
0720 NELSON, NANCY			X	X			2
0730 OLSEN, OTTO				X			1
0750 PERRY, PAMELA			X	X			2
0760 PIERCE, PAUL			X	X			2
0770 PUTNAM, PATTY				X			1
0795 ROTH, RITA			X				1
0805 SCHAFFER, SAMUEL			X				1
0810 SCHMIDT, SUSAN				X			1
0830 SCOTT, STEVEN				X			1
0850 SMITH, SHARON			X				1
0860 STONE, SALLY			X				1

```
          WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

INSTRUCTIONAL GROUPING RECOMMENDATION - SUMMARY              PAGE   3
UNIT A  RND DEMONSTRATION ELEMENTARY SCHOOL              AS OF 07-19-76

                          S  S  S  S  S  S
                          A  B  C  C  D  D
                          0  0  0  0  0  0
STUDENT NO. -   NAME      1  2  1  2  1  2

0870 TAYLOR, TIMOTHY               X                              1

0890 WAGNER, WALTER                X                              1

    *** TOTALS ***        0  0 18 36  0  0                       54
```

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            WISCONSIN DESIGN FOR READING SKILL DEVELOPMENT

INSTRUCTIONAL GROUPING RECOMMENDATION - OMISSIONS             PAGE   1
UNIT A  RND DEMONSTRATION ELEMENTARY SCHOOL              AS OF 07-19-76

STUDENTS  NOT  INCLUDED  IN  THE  GROUPING  RECOMMENDATIONS  FOR  THE
FOLLOWING  SKILLS :

    SKILL SS-B-01 : PICTURE SYMBOLS
    SKILL SS-B-02 : PICTURE GRIDS
    SKILL SS-C-01 : NONPICTORIAL SYMBOLS
    SKILL SS-C-02 : COLOR KEYS
    SKILL SS-D-01 : POINT & LINE SYMBOLS
    SKILL SS-D-02 : CARDINAL DIRECTIONS

          ID #  NAME                      SKILLS RECOMMENDED

          0070  ANDERSON, ANDY            SS-C-07  SS-C-08  SS-C-09
          0150  BERG, BECKY               SS-C-04  SS-C-05  SS-C-06
          0170  BRIGGS, BRUCE             SS-C-04  SS-C-05  SS-C-06
          0205  CARLSON, CARL             SS-C-04  SS-C-06  SS-C-07
          0310  DAVIS, DAVID              SS-C-03  SS-C-04  SS-C-06
          0510  HUBBARD, HOPE             SS-C-03  SS-C-04  SS-C-05
          0540  JACKSON, JOHN             SS-C-03  SS-C-05  SS-C-06
          0560  JOHNSON, JERRY            SS-C-06  SS-C-07  SS-C-08
          0700  MEYER, MARK               SS-C-07  SS-C-08  SS-C-09
          0780  REED, ROBERT              SS-C-07  SS-C-08  SS-C-09
          0790  ROBERTS, RICHARD          SS-C-04  SS-C-05  SS-C-06
          0840  SIMON, SANDRA             SS-C-03  SS-C-04  SS-C-06
          0880  TYLER, THEODORE           SS-C-07  SS-C-08  SS-C-09
          0900  WEBSTER, WENDY            SS-C-06  SS-C-07  SS-C-08
```

APPENDIX G

QUESTIONNAIRE - TEACHER PERCEPTIONS OF THE COMPUTERIZED GROUPING PROCEDURE

Evaluation of Computerized Grouping Procedure

PART A

Listed below are some criteria which may be considered when evaluating computerized procedures designed to place students into groups for instructional purposes. Please indicate your assessment of their desirability when the computerized grouping procedure operates within the framework of IGE. Do this by circling the number which best indicates the degree of desirability you attach to each feature. Please make comments where you wish to elaborate on your response.

1. Teachers should be able to specify the number of groups to be formed.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

2. Teachers should be able to specify the exact size of each group to be formed (e.g., Group 1, 10 students; Group 2, 20 students, etc.).

Very desirable Undesirable

1 2 3 4 5

Comment: ____

3. Teachers should be able to specify the group's size as a range (e.g., Group 1, 10-15; Group 2, 20-25, etc.).

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

In questions 4-8, assume the instructional program comprises skills or objectives each of which have prerequisite skills or objectives.

4. The teacher should be able to specify what skill/objective is to be studied by each particular group.

Very desirable   Undesirable

1 — 2 — 3 — 4 — 5

Comment: ____

____

5. The teacher should be able to specify the set of skills/objectives from which the skills/objectives to be studied by each group will be selected.

Very desirable   Undesirable

1 — 2 — 3 — 4 — 5

Comment: ____

____

6. Teachers should be able to specify that more than one group will study the same skill/objective.

Very desirable   Undesirable

1 — 2 — 3 — 4 — 5

Comment: ____

____

7. Teachers should be able to specify that only one group will study the same skill/objective.

Very desirable Undesirable

1 2 3 4 5

Comment: ____________________

____

8. Teachers should be able to specify particular students who are to be placed in the same group.

Very desirable Undesirable

1 2 3 4 5

Comment: ____________________

____

9. Teachers should be able to specify particular students who are not to be placed in the same group.

Very desirable Undesirable

1 2 3 4 5

Comment: ____________________

____

10. Teachers should be able to specify learner characteristics on the basis of which groups are to be formed. The specified characteristics may differ for different instructional purposes.

Very desirable Undesirable

1 2 3 4 5

Comment: ____________________

____

11. Teachers should be able to request groups which are maximally homogeneous according to the chosen learner characteristic(s) (e.g., learning style, prior achievement, etc.).

Very desirable Undesirable

1 2 3 4 5

Comments: ____

____

12. Teachers should be able to readily observe any differences existing between the groups on the chosen learner characteristics.

Very desirable Undesirable

1 2 3 4 5

Comments: ____

____

13. Teachers should be able to readily observe any similarities existing within the groups in the chosen learner characteristics.

Very desirable Undesirable

1 2 3 4 5

Comments: ____

____

14. The similarities on the chosen learner characteristics should be helpful to teachers when they are preparing instructional prescriptions.

Very desirable Undesirable

1 2 3 4 5

Comments: ____

____

15. The format in which the groups are presented to teachers should include:

(i) Names of students in alphabetical order

Very desirable Undesirable

1 2 3 4 5

(ii) The number of students in the group

Very desirable Undesirable

1 2 3 4 5

(iii) The skill to be taught to that group

Very desirable Undesirable

1 2 3 4 5

(iv) The average of each learner characteristic for the group

Very desirable Undesirable

1 2 3 4 5

Comment: ______________________________

____

16. The number of students omitted from groups because of ineligibility (on account of their mastering all skills/objectives considered or not meeting prerequisites) should be the minimum possible.

Very desirable Undesirable

1 2 3 4 5

Comment: ______________________________

____

17. Students omitted from groups should be shown in a separate group with the reasons for their omission shown.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

18. Alternative recommendations for grouping should be made for those students omitted from groups.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

19. Teachers should be able to request groups formed only on the basis of eligibility (based on mastery of prerequisites and non-mastery of specified skill/objective) and without reference to learner characteristics.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

20. Teachers should be able to request groups formed only on the basis of learner characteristics and without reference to eligibility.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

21. Teachers should be able to request groups formed from only a subset of the students of the unit when this is required.

Very desirable Undesirable

1 2 3 4 5

Comment: ____

____

22. The grouping procedure should be more efficient (take less staff time) than a manual grouping procedure (e.g., using McBee cards).

Very desirable | Undesirable

1 — 2 — 3 — 4 — 5

Comment: ____

____

23. The grouping procedure should be more efficient (take less staff time) than a CMI grouping procedure using Instructional Grouping Recommendation Forms.

Very desirable | Undesirable

1 — 2 — 3 — 4 — 5

Comment: ____

____

ARE THERE OTHER FEATURES OF A GROUPING PROCEDURE WHICH YOU CONSIDER DESIRABLE?

Comments: ____

____

____

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____

____

____

____

PART B

Listed below are the same 23 criteria. This time, however, please indicate the extent to which you perceive the computerized grouping procedure as meeting these criteria.

THE COMPUTERIZED GROUPING PROCEDURE YOU ARE EVALUATING
ALLOWS/PROVIDES TEACHERS:

1. to specify the number of groups to be formed

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

____

2. to specify the exact size of each group

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

____

3. to specify the group's size as a range

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

____

4. to specify the skill/objective to be studied by each particular group

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

____

5. to specify a set of skills/objectives from which the skills to be studied are chosen

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

6. to specify that more than one group will study the same skill/objective

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

7. to specify that only one group will study the same skill/objective

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

8. to specify that particular students must be placed in the same group

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____________________

9. to specify that particular students must not be placed in the same group

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

10. to specify learner characteristics on which to form groups

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

11. to obtain maximally homogeneous groups

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

12. to readily observe any differences existing between groups

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

13. to readily observe any similarities existing within the groups

Very successfully Unsuccessfully

1 2 3 4 5

14. to base instructional prescriptions on those similarities in learner characteristics possessed by each group

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

15. with information on group membership in an appropriate format

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

16. with an acceptably low number of omissions from groups

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

17. with a separate group of omitted students with reasons for their omissions shown

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ______________________________

____

18. with alternative recommendations for students omitted from groups

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

19. to request groups formed only on the basis of eligibility and without reference to learner characteristics

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

20. to request groups formed only on the basis of learner characteristics and without reference to eligibility

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

21. to request groups formed from a subset of the students of the unit.

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

22. to spend less time on grouping students than a manual grouping procedure using McBee cards

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

23. to spend less time on grouping students than a CMI grouping procedure using Instructional Grouping Recommendation Forms.

Very successfully Unsuccessfully

1 2 3 4 5

Comment: ____

____

ARE YOU ABLE TO NOTE ANY DIFFERENCES BETWEEN THE GROUPS FORMED BY TEACHERS AND THE GROUPS FORMED USING THE COMPUTERIZED PROCEDURE?

Comments: ____

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APPENDIX H

COMPUTER PRINTOUT FOR TEST 3, DMP GROUPING

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GROUPING ALGORITHM A
--------------------

NUMBER OF GROUPS REQUESTED =  5
NUMBER OF SKILLS CONSIDERED =  6
NUMBER OF LEARNER VARIABLES CONSIDERED =  2
NUMBER OF STUDENTS =  88

ELIGIBILITY FOR SKILLS TAKEN INTO ACCOUNT
LEARNER VARIABLES TAKEN INTO ACCOUNT
MORE THAN ONE GROUP MAY STUDY THE SAME SKILL

GROUP RANGES =  15 25   15 25   15 25   15 25   15 25

SKILLS CONSIDERED =  10 11 12 13 14 15

LEARNER VARIABLES CONSIDERED =   2  4

                                          STUDENT RECORDS

        STUDENT NAME          VL  VN  AC  AN  KT  SI  SG  EO  EW    SD    SM   SEX 1 2 3 4 5 6 7 8 91011121314

                              26. 38. 28. 36. 34. 36. 28. 38. 20.   4.0   23.   1. 0 1 1 0 0 1 0 0 0 0 0 0 0 0
 1360                         20. 32. 30. 30. 18. 30. 34. 18. 24.   3.4   19.   0. 0 0 0 0 0 0 0 0 0 1 0 0 1 0
 1617                         28. 26. 32. 22. 30. 38. 24. 30. 18.   4.4   25.   1. 0 0 0 0 0 1 0 0 0 0 0 0 1 0
 3075                         36. 20. 34. 32. 16. 20. 26. 34. 18.   9.5   29.   1. 0 0 1 0 0 0 0 1 0 0 1 1 1 1
 1830                         38. 38. 28. 22. 36. 34. 24. 26. 38.   4.0   12.   1. 1 0 0 0 0 0 0 1 0 1 0 0 1 0
 1375                         22. 34. 26. 26. 24. 30. 22. 24. 26.   5.0   22.   0. 1 0 0 0 0 0 0 0 0 0 1 1 0 0
 2130                         38. 34. 22. 14. 20. 34. 24. 28. 28.   8.5   33.   1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 1380                         30. 40. 36. 32. 34. 40. 32. 36. 34.  -1.0   26.   0. 0 0 0 1 0 0 0 0 0 0 1 1 1 1
 2400                         30. 36. 34. 32. 32. 34. 26. 34. 28.   4.0   11.   0. 0 0 0 0 0 0 1 0 0 1 1 0 1 1
 2640                         28. 28. 34. 28. 28. 32. 32. 20. 28.  -1.0   16.   1. 1 0 0 0 0 0 0 0 0 0 1 0 0 0
 1650                         16. 30. 30. 24. 28. 32. 22. 22. 18.   2.8   19.   1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 1390                         28. 40. 36. 28. 32. 32. 30. 30. 22.   3.7   25.   1. 0 0 1 0 0 0 0 1 0 1 1 0 1 0
 2140                         30. 32. 26. 24. 26. 26. 24. 30. 24.   3.4   19.   0. 0 0 1 0 0 0 1 0 0 0 0 1 0 0
 1660                         30. 32. 28. 28. 26. 34. 28. 30. 20.   3.4   17.   1. 0 0 0 0 0 0 0 1 0 1 0 0 0 0
 1400                         34. 26. 34. 32. 30. 20. 36. 26. 32.   5.0   25.   0. 0 1 0 0 0 0 0 0 0 0 0 1 0 0
 3100                         36. 38. 26. 32. 26. 32. 30. 28. 32.   3.1   11.   0. 1 0 0 0 0 0 0 0 0 1 0 0 1 0
 1405                         28. 32. 30. 32. 32. 28. 36. 32. 30.   4.0   21.   1. 0 0 0 0 0 0 0 0 0 1 1 0 0 0
 1900                         16. 34. 30. 34. 32. 34. 20. 16. 34.   3.4   21.   0. 0 0 0 0 0 0 1 0 0 0 0 0 1 1
 2440                         22. 34. 36. 32. 28. 26. 28. 38. 18.   4.4   32.   1. 0 0 0 1 0 0 0 1 0 0 0 1 1 1
 2450                         30. 32. 28. 30. 34. 34. 24. 36. 26.   2.9   14.   1. 0 0 0 0 0 1 1 0 0 1 0 0 0 0
 1410                         30. 34. 34. 30. 32. 38. 30. 24. 32.   4.4   26.   0. 0 0 0 0 0 0 0 1 0 0 0 0 1 1
 2720                         22. 26. 32. 32. 22. 28. 22. 28. 26.   3.4   20.   1. 1 0 0 0 0 0 0 0 0 0 0 0 0 0
 1420                         32. 30. 38. 32. 32. 26. 26. 30. 32.   4.0   19.   0. 1 0 0 0 0 0 0 1 0 0 0 1 1 1
 3135                         28. 20. 34. 20. 32. 28. 26. 34. 30.   4.5   23.   0. 0 1 0 0 1 1 0 0 0 0 0 1 1 1
 2740                         30. 22. 40. 32. 32. 40. 20. 18. 22.   4.3   15.   1. 1 0 0 0 0 0 0 0 0 1 1 0 0 0
 1440                         40. 40. 36. 38. 34. 40. 38. 36. 38.   4.4   22.   1. 1 0 0 0 0 0 1 0 0 1 0 0 0 0
 1910                         30. 22. 32. 30. 30. 28. 30. 32. 26.  -1.0   13.   1. 1 0 0 0 0 0 0 0 0 0 0 0 0 0
 2170                         26. 30. 34. 32. 36. 26. 38. 36. 30.   4.2   15.   0. 0 0 0 0 0 0 0 0 0 1 0 1 0 0
 1450                         28. 28. 32. 32. 34. 22. 34. 32. 26.   3.7   20.   0. 0 0 0 0 0 1 1 0 0 0 0 1 0 0
 1940                         30. 30. 22. 28. 24. 26. 28. 24. 36.   3.4   23.   1. 0 0 0 0 0 0 0 0 0 0 1 1 1 0
 1690                         28. 32. 32. 30. 30. 36. 28. 38. 32.   7.2   24.   1. 0 0 1 0 0 0 0 0 0 0 1 1 1 1
 2180                         30. 40. 30. 28. 30. 32. 32. 38. 36.  -1.0   21.   0. 1 0 0 0 0 0 0 0 0 1 1 1 0 0
 1705                         10. 40. 30. 36. 32. 34. 14. 16. 34.   3.4   28.   0. 0 0 1 0 0 1 1 0 0 0 1 1 0 1
 1950                         14. 28. 14. 22. 34. 30. 20. 20. 16.   3.8   15.   1. 1 0 0 0 0 0 0 0 0 0 1 1 1 0
 1470
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2920     38. 38. 24. 14. 22. 40. 22. 32. 24. 11.6  42.  1. 0 0 0 1 0 0 0 0 0 0 0 0 1 0
2200     24. 24. 28. 30. 12. 14. 30. 26. 16.  3.7  25.  0. 0 0 1 0 0 0 0 0 1 1 0 0 1 0
1490     30. 24. 30. 26. 24. 32. 26. 32. 24.  6.7  22.  0. 0 0 1 0 0 0 0 0 1 0 0 0 0 0
1710     28. 40. 34. 30. 24. 22. 22. 30. 40.  3.4  13.  0. 1 0 0 0 0 0 0 0 0 1 0 0 0 0
1720     34. 34. 38. 24. 36. 32. 26. 26. 26.  4.0  25.  1. 0 0 0 0 1 0 0 0 0 0 0 1 0 0
3190     28. 38. 30. 24. 34. 24. 36. 26. 34.  4.0  23.  0. 1 0 0 0 0 0 0 0 0 1 0 0 1 1
1970     16. 18. 28. 26. 20. 34. 10. 26. 18.  3.8  15.  1. 1 0 0 0 1 1 0 0 0 1 0 0 0 0
1980     20. 36. 38. 28. 34. 32. 28. 26. 24.  5.0  20.  0. 0 0 0 0 0 0 0 1 0 0 0 1 1 1
1730     30. 36. 40. 40. 32. 36. 28. 16. 28.  6.4  29.  1. 0 0 0 0 0 0 0 0 0 0 0 0 1 0
1740     34. 40. 38. 38. 30. 14. 36. 40. 34.  4.4  23.  1. 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1750     26. 20. 34. 32. 22. 36. 34. 30. 22.  3.4  16.  0. 0 0 0 0 0 0 0 0 0 1 0 0 0 0
1520     24. 34. 34. 30. 30. 40. 18. 24. 28.  4.8  27.  1. 0 1 1 0 1 0 0 0 0 0 0 1 1 0
3230     22. 30. 28. 20. 28. 22. 24. 28. 18.  6.0  27.  1. 1 0 0 0 0 0 0 1 0 0 1 0 0 0
2010     24. 36. 30. 36. 38. 30. 30. 28. 28.   .0  13.  1. 1 0 0 0 0 0 0 0 0 1 0 0 0 0
2020     22. 34. 28. 34. 36. 30. 30. 34. 32.  3.4  19.  1. 1 0 0 0 1 0 0 0 0 1 1 0 1 0
1500     20. 20. 22. 22. 30. 10. 24. 24. 26.  3.4  22.  1. 1 0 0 0 0 1 1 0 0 1 0 0 0 0
2230     34. 32. 36. 34. 30. 28. 26. 34. 32.  2.9  18.  1. 1 0 0 0 0 0 0 0 0 1 1 0 0 0
2240     22. 30. 34. 32. 28. 38. 26. 26. 30.  3.7  31.  1. 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1760     36. 34. 26. 28. 26. 40. 16. 22. 22.  5.8  31.  0. 0 0 1 1 0 0 0 0 1 0 1 1 1 0
1770     24. 38. 24. 26. 22. 34. 26. 20. 32.  4.1  25.  1. 0 0 1 0 0 0 0 0 0 0 1 1 1 1
1780     34. 38. 22. 20. 32. 34. 24. 28. 30.  3.7  22.  0. 0 0 0 0 1 0 0 0 0 0 1 1 1 0
2250     34. 34. 30. 24. 40. 34. 16. 22. 30.  3.7  14.  0. 1 0 0 0 0 0 0 0 0 1 0 0 0 0
2260     22. 34. 26. 28. 26. 28. 32. 22. 30.  4.5  22.  1. 0 1 1 0 0 0 0 1 0 1 0 0 0 0
3260     32. 24. 24. 24. 22. 36. 26. 30. 24.  6.0  37.  1. 0 0 0 0 1 0 0 0 0 0 0 1 1 0
1790     30. 28. 30. 26. 32. 28. 20. 26. 18.  3.4  21.  1. 1 0 0 0 0 0 0 0 0 0 0 0 0 0
2270     26. 32. 36. 26. 24. 26. 30. 12. 28.  3.0  15.  1. 1 0 0 0 0 0 0 0 0 1 1 0 1 0
1800     20. 28. 34. 28. 26. 36. 32. 34. 34.  2.6  10.  1. 1 0 0 0 1 1 1 0 0 1 0 0 0 0
1550     40. 36. 36. 34. 30. 34. 28. 32. 28.  3.0  13.  0. 1 0 0 0 0 0 0 0 0 0 1 0 1 0
1560     18. 30. 34. 32. 30. 30. 18. 26. 36.  2.2  14.  1. 1 0 0 0 0 0 0 0 0 1 0 0 0 0
1570     32. 40. 32. 28. 36. 32. 38. 38. 24.  4.0  28.  1. 0 1 1 0 0 0 0 1 0 0 0 0 0 0
1580     24. 28. 24. 30. 30. 28. 32. 30. 32.  3.7  25.  1. 0 0 0 0 0 0 0 0 0 0 0 1 0 0
2300     34. 30. 24. 28. 34. 38. 22. 36. 24.  3.0  21.  1. 1 0 0 0 0 1 0 0 0 1 1 0 0 0
2310     26. 28. 22. 24. 22. 28. 28. 32. 30.  6.4  25.  0. 0 0 1 0 0 0 0 0 1 0 0 1 0 0
2040     18. 36. 38. 28. 28. 18. 32. 36. 32.  3.4  23.  0. 0 0 0 0 1 1 0 0 0 0 1 1 1 0
 775     28. 36. 28. 28. 30. 40. 34. 34. 30.  3.4  19.  0. 0 0 0 0 0 0 0 0 0 1 1 0 0 0
3000     28. 36. 32. 32. 30. 38. 28. 32. 32.  5.7  43.  1. 0 0 0 1 0 0 0 0 0 1 0 0 0 0
3010     26. 32. 34. 30. 36. 40. 26. 28. 34.  4.4  31.  0. 0 0 0 0 0 0 0 0 0 1 0 0 0 0
2320     28. 30. 28. 30. 28. 24. 32. 26. 34.  2.8  20.  0. 0 0 0 0 0 1 0 0 0 0 0 1 0 0
3025     26. 34. 24. 24. 32. 28. 22. 30. 26. -1.0  23.  0. 1 0 0 0 1 1 0 0 0 1 0 0 0 0
2080     30. 36. 30. 26. 24. 22. 32. 24. 28.  7.8  27.  0. 0 0 0 0 0 0 0 0 0 0 1 1 1 1
 793     26. 32. 30. 34. 24. 28. 22. 20. 34.  4.0  15.  0. 0 0 0 0 0 0 0 1 0 0 0 0 0 1
2330     34. 32. 24. 34. 28. 32. 32. 28. 36.  4.4  23,  0. 0 0 0 0 0 0 0 1 0 0 0 0 1 1
1830     24. 26. 20. 24. 28. 22. 18. 22. 36.  4.4  18.  1. 1 0 0 0 1 1 0 0 0 0 0 0 1 0
2340     36. 40. 32. 36. 28. 34. 24. 34. 30.  4.4  14.  1. 1 0 0 0 1 0 0 0 0 1 0 0 0 0
1860     20. 34. 28. 22. 24. 28. 26. 24. 28.  5.3  30.  1. 0 0 0 0 1 0 0 0 0 0 0 1 1 0
2370     28. 34. 32. 20. 20. 40. 22. 28. 32.  5.8  30.  0. 0 1 1 0 0 0 0 0 1 0 0 0 0 0
3300     24. 32. 34. 36. 36. 40. 26. 40. 36.  2.6  10.  0. 0 0 0 0 0 0 0 0 0 1 0 0 0 0
1870     12. 24. 34. 22. 32. 32. 24. 28. 30.  4.4  26.  0. 0 0 1 0 0 0 0 1 0 0 0 0 1 1
1610     26. 34. 34. 34. 30. 34. 30. 34. 28.  3.4  25.  0. 0 1 1 0 0 0 0 1 0 1 1 0 1 0
2110     26. 36. 38. 34. 36. 30. 40. 36. 30.  2.9  21.  1. 0 0 1 0 0 1 1 0 0 0 0 1 0 1
1875     30. 28. 30. 30. 28. 24. 28. 32. 22.  7.8  22.  1. 0 0 0 0 1 0 0 0 0 0 0 1 1 1
2615     38. 28. 26. 30. 24. 32. 28. 28. 26.  3.0  32.  0. 0 0 0 0 0 0 0 1 0 1 0 0 1 1
1615     22. 32. 30. 28. 26. 28. 26. 30. 22. -1.0  18.  1. 1 0 0 0 0 0 0 0 0 0 1 0 0 1
2380     14. 26. 22. 26. 30. 32. 24. 28. 24.  2.7  15.  1. 0 0 0 0 0 0 0 0 0 1 0 0 0 0

NUMBER OF STUDENTS ELIGIBLE FOR THE FOLLOWING SKILLS -
TOPIC 49            --  36
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459

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TOPIC 54              --  27
TOPIC 55              --  30
TOPIC 56              --  38
TOPIC 58              --  21
TOPIC 59              --   0

 *** LEARNER VARIABLES - MEAN, VAR, AND STD. DEV. ***
VISUAL NUMERICAL    HAS MEAN =  31.02  VAR. =     34.24  SD. =   [illegible]
AUDITORY NUMERICAL HAS MEAN =  28.60  VAR. =     25.35  SD. =   [illegible]

 *** STANDARDIZED LEARNER VARIABLES ***
1360                                 1.12  1.45
1617                                  .03   .26
3075                                -1.06 -1.33
1630                                -2.15   .66
1375                                 1.12 -1.33
8130                                  .40  -.53
1380                                  .40 -2.92
8400                                 1.49   .66
2640                                  .76   .66
1650                                 -.69  -.14
1390                                 -.33  -.93
8140                                 1.49  -.14
1660                                  .03  -.93
1400                                  .03  -.14
3100                                -1.06   .66
1405                                 1.12   .66
1900                                  .03   .66
8440                                  .40  1.06
8450                                  .40   .66
1410                                  .03   .26
2720                                  .40   .26
1420                                -1.06   .66
3135                                 -.33   .66
2740                                -2.15 -1.72
1440                                -1.79   .66
1910                                 1.49  1.85
8170                                -1.79   .26
1450                                 -.33   .66
1940                                 -.69   .66
1690                                 -.33  -.14
8180                                  .03   .26
1705                                 1.49  -.14
1950                                 1.49  1.06
1470                                 -.69 -1.33
2920                                 1.12 -2.92
8200                                -1.42   .26
1490                                -1.42  -.53
1710                                 1.49   .26
1720                                  .40  -.93
2190                                 1.12  -.93
1970                                -2.51  -.53
1980                                  .76  -.14
1730                                  .76  2.25
1740                                 1.49  1.85
1780                                -2.15   .66
1880                                  .40   .26
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3230                                  -.33 -1.72
2010                                   .76  1.45
2020                                   .40  1.06
1500                                 -2.15 -1.33
2230                                   .03  1.06
2240                                  -.33   .66
1760                                   .40  -.14
1770                                  1.12  -.53
1780                                  1.12 -1.72
2250                                   .40  -.93
2260                                   .40  -.14
3260                                 -1.42  -.93
1790                                  -.69  -.53
2270                                   .03  -.53
1800                                  -.69  -.14
1550                                   .76  1.06
1560                                  -.33   .66
1570                                  1.49  -.14
1580                                  -.69   .26
2300                                  -.33  -.14
2310                                  -.69  -.93
2040                                   .76  -.14
 775                                   .76  -.14
3000                                   .76   .66
3010                                   .03   .26
2320                                  -.33   .26
3025                                   .40  -.93
2080                                   .76  -.53
 793                                   .03  1.06
2330                                   .03  1.06
1830                                 -1.05  -.93
2340                                  1.49  1.45
1860                                   .40 -1.33
2370                                   .40 -1.72
3300                                   .03  1.45
1870                                 -1.42 -1.33
1610                                   .40  1.06
2110                                   .76  1.06
1875                                  -.69   .26
2615                                  -.69   .26
1615                                   .03  -.14
2380                                 -1.42  -.53

SKILLS ASSIGNED TO GROUPS -   4  1  3  2  4

SKILL # 4, 1ST STUDENT =  35, 2ND STUDENT =  43, DIFFERENCE =      5.176
      .000   35       1.192   55       1.589    5       1.747   79       1.986   40
     2.383   54       2.411   74       2.415   34       2.621   60       2.699    3
     2.804   12       2.806   42       2.804   68       2.874   53       2.951   77
     3.001   82       3.138   30       3.229   58       3.260   21       3.260   46
     3.360    2       3.360   31       3.483   24       3.575   16       3.593    8
     3.593    9       3.648   19       3.661   85       3.661   86       3.840   23
     3.989   62       4.038   18       4.038   49       4.038   83       4.072   36
     4.119   76       4.847    4       5.176   43

 *** PARTITIONING FOR SKILL # 4 ***
COMPUTED PARTITION WITH  38 STUDENTS:  19   19
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BALANCED PARTITION WITH  88 STUDENTS:  18   19

*** MEAN OF EACH GROUP ***
GROUP # 1   -.084    .506
GROUP # 2    .063    .229
GROUP # 3    .021   -.135
GROUP # 4    .276    .824
GROUP # 5    .244   -.846

*** STUDENT GROUP ASSIGNMENTS ***
     STU #  1 ASG TO  0 W. DIST   .00     STU #  2 ASG TO  2 W. DIST   .04     STU #  3 ASG TO  5 W. DIST  1.39
     STU #  4 ASG TO  1 W. DIST  2.13     STU #  5 ASG TO  5 W. DIST  1.00     STU #  6 ASG TO  3 W. DIST   .58
     STU #  7 ASG TO  0 W. DIST   .00     STU #  8 ASG TO  4 W. DIST  1.37     STU #  9 ASG TO  1 W. DIST   .79
     STU # 10 ASG TO  4 W. DIST   .98     STU # 11 ASG TO  0 W. DIST   .00     STU # 12 ASG TO  4 W. DIST  1.82
     STU # 13 ASG TO  3 W. DIST   .79     STU # 14 ASG TO  2 W. DIST   .37     STU # 15 ASG TO  3 W. DIST  1.34
     STU # 16 ASG TO  2 W. DIST  1.14     STU # 17 ASG TO  2 W. DIST   .43     STU # 18 ASG TO  1 W. DIST   .68
     STU # 19 ASG TO  1 W. DIST   .43     STU # 20 ASG TO  2 W. DIST   .04     STU # 21 ASG TO  1 W. DIST   .84
     STU # 22 ASG TO  0 W. DIST   .00     STU # 23 ASG TO  1 W. DIST   .31     STU # 24 ASG TO  5 W. DIST  2.55
     STU # 25 ASG TO  2 W. DIST  1.90     STU # 26 ASG TO  2 W. DIST  2.16     STU # 27 ASG TO  0 W. DIST   .00
     STU # 28 ASG TO  2 W. DIST   .58     STU # 29 ASG TO  3 W. DIST  1.07     STU # 30 ASG TO  3 W. DIST   .35
     STU # 31 ASG TO  4 W. DIST   .34     STU # 32 ASG TO  4 W. DIST  1.22     STU # 33 ASG TO  4 W. DIST  1.99
     STU # 34 ASG TO  5 W. DIST  1.05     STU # 35 ASG TO  5 W. DIST  2.25     STU # 36 ASG TO  1 W. DIST  1.44
     STU # 37 ASG TO  0 W. DIST   .00     STU # 38 ASG TO  2 W. DIST  1.42     STU # 39 ASG TO  3 W. DIST   .88
     STU # 40 ASG TO  5 W. DIST   .88     STU # 41 ASG TO  2 W. DIST  2.69     STU # 42 ASG TO  3 W. DIST   .74
     STU # 43 ASG TO  1 W. DIST  1.83     STU # 44 ASG TO  0 W. DIST   .00     STU # 45 ASG TO  2 W. DIST  2.25
     STU # 46 ASG TO  1 W. DIST   .54     STU # 47 ASG TO  4 W. DIST  1.85     STU # 48 ASG TO  2 W. DIST  1.41
     STU # 49 ASG TO  1 W. DIST   .62     STU # 50 ASG TO  2 W. DIST  2.70     STU # 51 ASG TO  2 W. DIST   .83
     STU # 52 ASG TO  0 W. DIST   .00     STU # 53 ASG TO  4 W. DIST   .20     STU # 54 ASG TO  5 W. DIST   .93
     STU # 55 ASG TO  5 W. DIST  1.24     STU # 56 ASG TO  2 W. DIST  1.21     STU # 57 ASG TO  2 W. DIST   .49
     STU # 58 ASG TO  3 W. DIST  1.65     STU # 59 ASG TO  0 W. DIST   .00     STU # 60 ASG TO  5 W. DIST   .38
     STU # 61 ASG TO  2 W. DIST   .84     STU # 62 ASG TO  1 W. DIST   .91     STU # 63 ASG TO  2 W. DIST   .90
     STU # 64 ASG TO  0 W. DIST   .00     STU # 65 ASG TO  3 W. DIST   .82     STU # 66 ASG TO  2 W. DIST   .54
     STU # 67 ASG TO  3 W. DIST  1.07     STU # 68 ASG TO  4 W. DIST   .51     STU # 69 ASG TO  4 W. DIST   .81
     STU # 70 ASG TO  2 W. DIST   .82     STU # 71 ASG TO  2 W. DIST   .04     STU # 72 ASG TO  3 W. DIST   .53
     STU # 73 ASG TO  2 W. DIST  1.21     STU # 74 ASG TO  5 W. DIST   .60     STU # 75 ASG TO  0 W. DIST   .00
     STU # 76 ASG TO  1 W. DIST   .46     STU # 77 ASG TO  5 W. DIST  1.30     STU # 78 ASG TO  2 W. DIST  1.08
     STU # 79 ASG TO  5 W. DIST   .50     STU # 80 ASG TO  0 W. DIST   .00     STU # 81 ASG TO  2 W. DIST  1.80
     STU # 82 ASG TO  5 W. DIST  1.73     STU # 83 ASG TO  1 W. DIST   .62     STU # 84 ASG TO  3 W. DIST  1.48
     STU # 85 ASG TO  1 W. DIST   .75     STU # 86 ASG TO  1 W. DIST   .75     STU # 87 ASG TO  4 W. DIST   .20
     STU # 88 ASG TO  2 W. DIST  1.67     STU #

FOR ITERATION # 1 TOTAL DISTANCE =  78.326 WHICH DIFFERS FROM PREVIOUS BY   78.326
    TOTAL SUM OF SQUARES WITHIN =   111.17,  MEAN SUM OF SQUARES WITHIN =     1.34

*** MEAN OF EACH GROUP ***
GROUP # 1   -.040    .765
GROUP # 2   -.127    .325
GROUP # 3   -.240   -.135
GROUP # 4    .628   -.063
GROUP # 5    .033  -1.266

*** STUDENT GROUP ASSIGNMENTS ***
     STU #  1 ASG TO  0 W. DIST   .00     STU #  2 ASG TO  2 W. DIST   .17     STU #  3 ASG TO  5 W. DIST  1.09
     STU #  4 ASG TO  3 W. DIST  2.67     STU #  5 ASG TO  5 W. DIST  1.60     STU #  6 ASG TO  4 W. DIST   .58
     STU #  7 ASG TO  0 W. DIST   .00     STU #  8 ASG TO  4 W. DIST  1.12     STU #  9 ASG TO  4 W. DIST   .79
     STU # 10 ASG TO  4 W. DIST  1.32     STU # 11 ASG TO  0 W. DIST   .00     STU # 12 ASG TO  4 W. DIST   .06
     STU # 13 ASG TO  3 W. DIST   .84     STU # 14 ASG TO  2 W. DIST   .45     STU # 15 ASG TO  3 W. DIST  1.14
     STU # 16 ASG TO  1 W. DIST  1.17      STU # 17 ASG TO  2 W. DIST   .37      STU # 18 ASG TO  1 W. DIST   .52
     STU # 19 ASG TO  1 W. DIST   .45      STU # 20 ASG TO  2 W. DIST   .17      STU # 21 ASG TO  1 W. DIST   .67
     STU # 22 ASG TO  0 W. DIST   .00      STU # 23 ASG TO  1 W. DIST   .31      STU # 24 ASG TO  5 W. DIST  2.23
     STU # 25 ASG TO  2 W. DIST  1.69      STU # 26 ASG TO  2 W. DIST  2.22      STU # 27 ASG TO  0 W. DIST   .00
     STU # 28 ASG TO  2 W. DIST   .39      STU # 29 ASG TO  3 W. DIST   .92      STU # 30 ASG TO  3 W. DIST   .09
     STU # 31 ASG TO  3 W. DIST   .68      STU # 32 ASG TO  4 W. DIST   .86      STU # 33 ASG TO  4 W. DIST  1.41
     STU # 34 ASG TO  5 W. DIST   .73      STU # 35 ASG TO  5 W. DIST  1.98      STU # 36 ASG TO  2 W. DIST  1.30
     STU # 37 ASG TO  0 W. DIST   .00      STU # 38 ASG TO  2 W. DIST  1.62      STU # 39 ASG TO  3 W. DIST  1.02
     STU # 40 ASG TO  5 W. DIST  1.16      STU # 41 ASG TO  2 W. DIST  2.54      STU # 42 ASG TO  3 W. DIST  1.00
     STU # 43 ASG TO  1 W. DIST  1.68      STU # 44 ASG TO  0 W. DIST   .00      STU # 45 ASG TO  2 W. DIST  2.05
     STU # 46 ASG TO  1 W. DIST   .67      STU # 47 ASG TO  4 W. DIST  1.92      STU # 48 ASG TO  2 W. DIST  1.44
     STU # 49 ASG TO  1 W. DIST   .52      STU # 50 ASG TO  2 W. DIST  2.61      STU # 51 ASG TO  2 W. DIST   .75
     STU # 52 ASG TO  0 W. DIST   .00      STU # 53 ASG TO  4 W. DIST   .24      STU # 54 ASG TO  4 W. DIST   .68
     STU # 55 ASG TO  5 W. DIST  1.18      STU # 56 ASG TO  2 W. DIST  1.36      STU # 57 ASG TO  2 W. DIST   .70
     STU # 58 ASG TO  3 W. DIST  1.49      STU # 59 ASG TO  0 W. DIST   .00      STU # 60 ASG TO  5 W. DIST   .73
     STU # 61 ASG TO  2 W. DIST   .73      STU # 62 ASG TO  1 W. DIST   .85      STU # 63 ASG TO  2 W. DIST   .30
     STU # 64 ASG TO  0 W. DIST   .00      STU # 65 ASG TO  3 W. DIST   .60      STU # 66 ASG TO  2 W. DIST   .50
     STU # 67 ASG TO  3 W. DIST   .92      STU # 68 ASG TO  4 W. DIST   .15      STU # 69 ASG TO  4 W. DIST   .15
     STU # 70 ASG TO  2 W. DIST   .95      STU # 71 ASG TO  2 W. DIST   .17      STU # 72 ASG TO  3 W. DIST   .41
     STU # 73 ASG TO  2 W. DIST  1.36      STU # 74 ASG TO  4 W. DIST   .49      STU # 75 ASG TO  0 W. DIST   .00
     STU # 76 ASG TO  1 W. DIST   .30      STU # 77 ASG TO  5 W. DIST  1.14      STU # 78 ASG TO  2 W. DIST  1.97
     STU # 79 ASG TO  5 W. DIST   .37      STU # 80 ASG TO  0 W. DIST   .00      STU # 81 ASG TO  2 W. DIST  1.16
     STU # 82 ASG TO  5 W. DIST  1.46      STU # 83 ASG TO  1 W. DIST   .52      STU # 84 ASG TO  3 W. DIST  1.56
     STU # 85 ASG TO  3 W. DIST   .60      STU # 86 ASG TO  2 W. DIST   .57      STU # 87 ASG TO  4 W. DIST   .60
     STU # 88 ASG TO  2 W. DIST  1.55      STU #

FOR ITERATION # 2 TOTAL DISTANCE =  74.152 WHICH DIFFERS FROM PREVIOUS BY    4.174
    TOTAL SUM OF SQUARES WITHIN =   100.09,  MEAN SUM OF SQUARES WITHIN =    1.21

 *** MEAN OF EACH GROUP ***
GROUP # 1    .430    .912
GROUP # 2  -.247    .308
GROUP # 3  -.435    .006
GROUP # 4    .709  -.135
GROUP # 5  -.132 -1.399

 *** STUDENT GROUP ASSIGNMENTS ***
     STU #  1 ASG TO  0 W. DIST   .00      STU #  2 ASG TO  2 W. DIST   .28      STU #  3 ASG TO  5 W. DIST   .93
     STU #  4 ASG TO  3 W. DIST  1.83      STU #  5 ASG TO  5 W. DIST  1.26      STU #  6 ASG TO  4 W. DIST   .50
     STU #  7 ASG TO  0 W. DIST   .00      STU #  8 ASG TO  1 W. DIST  1.09      STU #  9 ASG TO  1 W. DIST   .42
     STU # 10 ASG TO  4 W. DIST  1.40      STU # 11 ASG TO  0 W. DIST   .00      STU # 12 ASG TO  4 W. DIST   .78
     STU # 13 ASG TO  3 W. DIST  1.05      STU # 14 ASG TO  2 W. DIST   .52      STU # 15 ASG TO  3 W. DIST   .90
     STU # 16 ASG TO  1 W. DIST   .74      STU # 17 ASG TO  2 W. DIST   .45      STU # 18 ASG TO  1 W. DIST   .19
     STU # 19 ASG TO  1 W. DIST   .25      STU # 20 ASG TO  2 W. DIST   .28      STU # 21 ASG TO  1 W. DIST   .65
     STU # 22 ASG TO  0 W. DIST   .00      STU # 23 ASG TO  3 W. DIST   .66      STU # 24 ASG TO  5 W. DIST  2.06
     STU # 25 ASG TO  2 W. DIST  1.58      STU # 26 ASG TO  2 W. DIST  2.32      STU # 27 ASG TO  0 W. DIST   .00
     STU # 28 ASG TO  2 W. DIST   .36      STU # 29 ASG TO  3 W. DIST   .70      STU # 30 ASG TO  3 W. DIST   .18
     STU # 31 ASG TO  3 W. DIST   .53      STU # 32 ASG TO  4 W. DIST   .78      STU # 33 ASG TO  4 W. DIST  1.42
     STU # 34 ASG TO  5 W. DIST   .57      STU # 35 ASG TO  5 W. DIST  1.97      STU # 36 ASG TO  2 W. DIST  1.18
     STU # 37 ASG TO  0 W. DIST   .00      STU # 38 ASG TO  2 W. DIST  1.74      STU # 39 ASG TO  3 W. DIST  1.25
     STU # 40 ASG TO  5 W. DIST  1.34      STU # 41 ASG TO  2 W. DIST  2.42      STU # 42 ASG TO  1 W. DIST  1.10
     STU # 43 ASG TO  1 W. DIST  1.38      STU # 44 ASG TO  0 W. DIST   .00      STU # 45 ASG TO  2 W. DIST  1.93
     STU # 46 ASG TO  1 W. DIST   .65      STU # 47 ASG TO  4 W. DIST  1.90      STU # 48 ASG TO  2 W. DIST  1.53
     STU # 49 ASG TO  1 W. DIST   .15      STU # 50 ASG TO  2 W. DIST  2.51      STU # 51 ASG TO  2 W. DIST   .80
     STU # 52 ASG TO  0 W. DIST   .00      STU # 53 ASG TO  4 W. DIST   .31      STU # 54 ASG TO  4 W. DIST   .57
     STU # 55 ASG TO  5 W. DIST  1.30      STU # 56 ASG TO  2 W. DIST  1.39      STU # 57 ASG TO  2 W. DIST   .78
     STU # 58 ASG TO  3 W. DIST  1.38      STU # 59 ASG TO  0 W. DIST   .00      STU # 60 ASG TO  4 W. DIST   .78
     STU = 61 ASG TO  2 W. DIST   .63      STU = 62 ASG TO  1 W. DIST   .36      STU = 63 ASG TO  2 W. DIST   .30
     STU = 64 ASG TO  0 W. DIST   .00      STU = 65 ASG TO  3 W. DIST   .30      STU = 66 ASG TO  2 W. DIST   .65
     STU = 67 ASG TO  3 W. DIST   .97      STU = 68 ASG TO  4 W. DIST   .05      STU = 69 ASG TO  4 W. DIST   .05
     STU = 70 ASG TO  2 W. DIST  1.07      STU = 71 ASG TO  2 W. DIST   .20      STU = 72 ASG TO  3 W. DIST   .28
     STU = 73 ASG TO  2 W. DIST  1.39      STU = 74 ASG TO  4 W. DIST   .40      STU = 75 ASG TO  0 W. DIST   .00
     STU = 76 ASG TO  1 W. DIST   .42      STU = 77 ASG TO  5 W. DIST  1.04      STU = 78 ASG TO  2 W. DIST  2.00
     STU = 79 ASG TO  5 W. DIST   .53      STU = 80 ASG TO  0 W. DIST   .00      STU = 81 ASG TO  2 W. DIST  1.10
     STU = 82 ASG TO  5 W. DIST  1.29      STU = 83 ASG TO  1 W. DIST   .15      STU = 84 ASG TO  3 W. DIST  1.59
     STU = 85 ASG TO  3 W. DIST   .36      STU = 86 ASG TO  2 W. DIST   .45      STU = 87 ASG TO  4 W. DIST   .60
     STU = 88 ASG TO  2 W. DIST  1.44      STU =

FOR ITERATION = 3 TOTAL DISTANCE =  70.856 WHICH DIFFERS FROM PREVIOUS BY    3.276
    TOTAL SUM OF SQUARES WITHIN =    94.64,  MEAN SUM OF SQUARES WITHIN =    1.14

 *** MEAN OF EACH GROUP ***
GROUP = 1    .621   .812
GROUP = 2  -.247    .308
GROUP = 3  -.512    .063
GROUP = 4    .593  -.288
GROUP = 5  -.149 -1.486

 *** STUDENT GROUP ASSIGNMENTS ***
     STU =  1 ASG TO  0 W. DIST   .00      STU =  2 ASG TO  2 W. DIST   .28      STU =  3 ASG TO  5 W. DIST   .92
     STU =  4 ASG TO  3 W. DIST  1.74      STU =  5 ASG TO  5 W. DIST  1.28      STU =  6 ASG TO  4 W. DIST   .31
     STU =  7 ASG TO  0 W. DIST   .00      STU =  8 ASG TO  1 W. DIST   .88      STU =  9 ASG TO  1 W. DIST   .21
     STU = 10 ASG TO  4 W. DIST  1.30      STU = 11 ASG TO  0 W. DIST   .00      STU = 12 ASG TO  4 W. DIST   .91
     STU = 13 ASG TO  3 W. DIST  1.13      STU = 14 ASG TO  2 W. DIST   .52      STU = 15 ASG TO  3 W. DIST   .81
     STU = 16 ASG TO  1 W. DIST   .53      STU = 17 ASG TO  2 W. DIST   .45      STU = 18 ASG TO  1 W. DIST   .33
     STU = 19 ASG TO  1 W. DIST   .27      STU = 20 ASG TO  2 W. DIST   .28      STU = 21 ASG TO  1 W. DIST   .09
     STU = 22 ASG TO  0 W. DIST   .00      STU = 23 ASG TO  3 W. DIST   .62      STU = 24 ASG TO  5 W. DIST  2.01
     STU = 25 ASG TO  2 W. DIST  1.58      STU = 26 ASG TO  2 W. DIST  2.32      STU = 27 ASG TO  0 W. DIST   .00
     STU = 28 ASG TO  2 W. DIST   .36      STU = 29 ASG TO  3 W. DIST   .62      STU = 30 ASG TO  3 W. DIST   .87
     STU = 31 ASG TO  3 W. DIST   .58      STU = 32 ASG TO  4 W. DIST   .91      STU = 33 ASG TO  4 W. DIST  1.02
     STU = 34 ASG TO  5 W. DIST   .57      STU = 35 ASG TO  5 W. DIST  1.91      STU = 36 ASG TO  2 W. DIST  1.10
     STU = 37 ASG TO  0 W. DIST   .00      STU = 38 ASG TO  2 W. DIST  1.74      STU = 39 ASG TO  3 W. DIST  1.35
     STU = 40 ASG TO  5 W. DIST  1.39      STU = 41 ASG TO  2 W. DIST  2.42      STU = 42 ASG TO  1 W. DIST   .90
     STU = 43 ASG TO  1 W. DIST  1.44      STU = 44 ASG TO  0 W. DIST   .00      STU = 45 ASG TO  2 W. DIST  1.03
     STU = 46 ASG TO  1 W. DIST   .59      STU = 47 ASG TO  4 W. DIST  1.71      STU = 48 ASG TO  2 W. DIST  1.83
     STU = 49 ASG TO  1 W. DIST   .33      STU = 50 ASG TO  2 W. DIST  2.51      STU = 51 ASG TO  2 W. DIST   .08
     STU = 52 ASG TO  0 W. DIST   .00      STU = 53 ASG TO  4 W. DIST   .25      STU = 54 ASG TO  4 W. DIST   .59
     STU = 55 ASG TO  5 W. DIST  1.30      STU = 56 ASG TO  2 W. DIST  1.39      STU = 57 ASG TO  2 W. DIST   .70
     STU = 58 ASG TO  3 W. DIST  1.35      STU = 59 ASG TO  0 W. DIST   .00      STU = 60 ASG TO  4 W. DIST   .41
     STU = 61 ASG TO  2 W. DIST   .63      STU = 62 ASG TO  1 W. DIST   .28      STU = 63 ASG TO  2 W. DIST   .36
     STU = 64 ASG TO  0 W. DIST   .00      STU = 65 ASG TO  3 W. DIST   .27      STU = 66 ASG TO  2 W. DIST   .65
     STU = 67 ASG TO  3 W. DIST  1.01      STU = 68 ASG TO  4 W. DIST   .23      STU = 69 ASG TO  4 W. DIST   .03
     STU = 70 ASG TO  2 W. DIST  1.07      STU = 71 ASG TO  2 W. DIST   .28      STU = 72 ASG TO  3 W. DIST   .27
     STU = 73 ASG TO  2 W. DIST  1.30      STU = 74 ASG TO  4 W. DIST   .30      STU = 75 ASG TO  0 W. DIST   .00
     STU = 76 ASG TO  1 W. DIST   .64      STU = 77 ASG TO  5 W. DIST  1.07      STU = 78 ASG TO  2 W. DIST  2.00
     STU = 79 ASG TO  5 W. DIST   .57      STU = 80 ASG TO  0 W. DIST   .00      STU = 81 ASG TO  2 W. DIST  1.10
     STU = 82 ASG TO  5 W. DIST  1.28      STU = 83 ASG TO  1 W. DIST   .33      STU = 84 ASG TO  3 W. DIST  1.61
     STU = 85 ASG TO  3 W. DIST   .27      STU = 86 ASG TO  2 W. DIST   .45      STU = 87 ASG TO  4 W. DIST   .50
     STU = 88 ASG TO  2 W. DIST  1.44      STU =

FOR ITERATION = 4 TOTAL DISTANCE =  70.527 WHICH DIFFERS FROM PREVIOUS BY     .329
    TOTAL SUM OF SQUARES WITHIN =    93.19,  MEAN SUM OF SQUARES WITHIN =    1.12

 *** MEAN OF EACH GROUP ***
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GROUP # 1   .421   .812
GROUP # 2  -.247   .308
GROUP # 3  -.312   .063
GROUP # 4   .593  -.288
GROUP # 5  -.149 -1.486

 *** STUDENT GROUP ASSIGNMENTS ***
     STU #  1 ASG TO  0 W. DIST   .00      STU #  2 ASG TO  2 W. DIST   .28      STU #  3 ASG TO  5 W. DIST   .92
     STU #  4 ASG TO  3 W. DIST  1.76      STU #  5 ASG TO  5 W. DIST  1.28      STU #  6 ASG TO  4 W. DIST   .31
     STU #  7 ASG TO  0 W. DIST   .00      STU #  8 ASG TO  1 W. DIST   .58      STU #  9 ASG TO  1 W. DIST   .21
     STU # 10 ASG TO  4 W. DIST  1.30      STU # 11 ASG TO  0 W. DIST   .00      STU # 12 ASG TO  4 W. DIST   .91
     STU # 13 ASG TO  3 W. DIST  1.13      STU # 14 ASG TO  2 W. DIST   .52      STU # 15 ASG TO  3 W. DIST   .81
     STU # 16 ASG TO  1 W. DIST   .53      STU # 17 ASG TO  2 W. DIST   .45      STU # 18 ASG TO  1 W. DIST   .33
     STU # 19 ASG TO  1 W. DIST   .27      STU # 20 ASG TO  2 W. DIST   .28      STU # 21 ASG TO  1 W. DIST   .59
     STU # 22 ASG TO  0 W. DIST   .00      STU # 23 ASG TO  3 W. DIST   .62      STU # 24 ASG TO  5 W. DIST  2.01
     STU # 25 ASG TO  2 W. DIST  1.58      STU # 26 ASG TO  2 W. DIST  2.32      STU # 27 ASG TO  0 W. DIST   .00
     STU # 28 ASG TO  2 W. DIST   .36      STU # 29 ASG TO  3 W. DIST   .62      STU # 30 ASG TO  3 W. DIST   .27
     STU # 31 ASG TO  3 W. DIST   .58      STU # 32 ASG TO  4 W. DIST   .91      STU # 33 ASG TO  4 W. DIST  1.62
     STU # 34 ASG TO  5 W. DIST   .57      STU # 35 ASG TO  5 W. DIST  1.91      STU # 36 ASG TO  2 W. DIST  1.18
     STU # 37 ASG TO  0 W. DIST   .00      STU # 38 ASG TO  2 W. DIST  1.74      STU # 39 ASG TO  3 W. DIST  1.35
     STU # 40 ASG TO  5 W. DIST  1.39      STU # 41 ASG TO  2 W. DIST  2.42      STU # 42 ASG TO  1 W. DIST   .96
     STU # 43 ASG TO  1 W. DIST  1.44      STU # 44 ASG TO  0 W. DIST   .00      STU # 45 ASG TO  2 W. DIST  1.93
     STU # 46 ASG TO  1 W. DIST   .59      STU # 47 ASG TO  4 W. DIST  1.71      STU # 48 ASG TO  2 W. DIST  1.53
     STU # 49 ASG TO  1 W. DIST   .33      STU # 50 ASG TO  2 W. DIST  2.51      STU # 51 ASG TO  2 W. DIST   .80
     STU # 52 ASG TO  0 W. DIST   .00      STU # 53 ASG TO  4 W. DIST   .25      STU # 54 ASG TO  4 W. DIST   .59
     STU # 55 ASG TO  5 W. DIST  1.30      STU # 56 ASG TO  2 W. DIST  1.39      STU # 57 ASG TO  2 W. DIST   .78
     STU # 58 ASG TO  3 W. DIST  1.35      STU # 59 ASG TO  0 W. DIST   .00      STU # 60 ASG TO  4 W. DIST   .61
     STU # 61 ASG TO  2 W. DIST   .63      STU # 62 ASG TO  1 W. DIST   .28      STU # 63 ASG TO  2 W. DIST   .36
     STU # 64 ASG TO  0 W. DIST   .00      STU # 65 ASG TO  3 W. DIST   .27      STU # 66 ASG TO  2 W. DIST   .45
     STU # 67 ASG TO  3 W. DIST  1.01      STU # 68 ASG TO  4 W. DIST   .23      STU # 69 ASG TO  4 W. DIST   .23
     STU # 70 ASG TO  2 W. DIST  1.07      STU # 71 ASG TO  2 W. DIST   .28      STU # 72 ASG TO  3 W. DIST   .27
     STU # 73 ASG TO  2 W. DIST  1.39      STU # 74 ASG TO  4 W. DIST   .30      STU # 75 ASG TO  0 W. DIST   .00
     STU # 76 ASG TO  1 W. DIST   .64      STU # 77 ASG TO  5 W. DIST  1.07      STU # 78 ASG TO  2 W. DIST  2.08
     STU # 79 ASG TO  5 W. DIST   .57      STU # 80 ASG TO  0 W. DIST   .00      STU # 81 ASG TO  2 W. DIST  1.18
     STU # 82 ASG TO  5 W. DIST  1.28      STU # 83 ASG TO  1 W. DIST   .33      STU # 84 ASG TO  3 W. DIST  1.61
     STU # 85 ASG TO  3 W. DIST   .27      STU # 86 ASG TO  2 W. DIST   .45      STU # 87 ASG TO  4 W. DIST   .58
     STU # 88 ASG TO  2 W. DIST  1.44      STU #

FOR ITERATION # 5 TOTAL DISTANCE =  70,527 WHICH DIFFERS FROM PREVIOUS BY     .000
    TOTAL SUM OF SQUARES WITHIN =    93.15,  MEAN SUM OF SQUARES WITHIN =    1.12

GRP #   SKILL   NO. IN GROUP
  1       4          13
  2       1          26
  3       3          14
  4       2          13
  5       4          10

 STUDENT # 50 HAS BEEN ROOTED OUT OF GRP # 2
1500 MCCARTHY, THOMAS          HAS BEEN REMOVED FROM GRP # 2
    TOTAL DIST. =  67,793 TSSW =    86,61 MSSW =    1.04
```

```
GROUP # 1, SKILL # 4  TOPIC 56
NUMBER OF STUDENTS RECOMMENDED =  13

STUD #              STUDENT NAME          DISTANCE

 2400                                       .881
 2640                                       .207
 1405                                       .526
 2440                                       .331
 2450                                       .271
 2720                                       .594
 1980                                       .957
 1730                                      1.443
 1520                                       .594
 2020                                       .331
 1550                                       .282
 2330                                       .636
 1610                                       .331

 *** LEARNER VARIABLES - MEAN, VAR, AND STD, DEV, ***
VISUAL NUMERICAL   HAS MEAN =   .82  VAR. =        .13  SD. =     .36
AUDITORY NUMERICAL HAS MEAN =   .81  VAR. =        .30  SD. =     .55
```

```
GROUP # 2, SKILL # 1  TOPIC 49
NUMBER OF STUDENTS RECOMMENDED =  25 -

STUD #              STUDENT NAME             DISTANCE

 1617                                           .232
 1400                                           .548
 1900                                           .351
 1410                                           .232
 1440                                          1.640
 1910                                          2.221
 1450                                           .328
 2200                                          1.256
 1710                                          1.662
 1970                                          2.511
 1750                                          1.999
 2010                                          1.426
 2230                                           .713
 2250                                          1.421
 2260                                           .762
 1800                                           .730
 1560                                           .328
 2300                                           .533
 3000                                           .974
 3010                                           .232
 3025                                          1.421
 2340                                          1.979
 3300                                          1.099
 2615                                           .535
 2380                                          1.544

 *** LEARNER VARIABLES - MEAN, VAR. AND STD. DEV. ***
VISUAL NUMERICAL   HAS MEAN =   -.17  VAR. =      1.08  SD. =   1.04
AUDITORY NUMERICAL HAS MEAN =    .37  VAR. =       .51  SD. =    .72
```

```
GROUP # 3, SKILL # 3  TOPIC 55
NUMBER OF STUDENTS RECOMMENDED =  14

STUD #             STUDENT NAME              DISTANCE

                                               1.742
  1630                                         1.133
  1660                                          .808
  3100                                          .623
  3135                                          .623
  1940                                          .269
  1690                                          .581
  2180                                         1.346
  1720                                         1.346
  3260                                          .269
  1580                                         1.010
  2310                                          .269
  2320                                         1.614
  2110                                          .269
  1875

  *** LEARNER VARIABLES - MEAN, VAR, AND STD. DEV. ***
 VISUAL NUMERICAL    HAS MEAN =    -.51  VAR. =          .51  SD. =     .71
 AUDITORY NUMERICAL  HAS MEAN =     .06  VAR. =          .67  SD. =     [illegible]
```

```
GROUP # 4, SKILL # 2  TOPIC 54
NUMBER OF STUDENTS RECOMMENDED =  13

STUD #               STUDENT NAME            DISTANCE

 2130                                          .313
 1650                                         1.296
 2140                                          .908
 1705                                          .908
 1950                                         1.615
 3230                                         1.707
 1760                                          .248
 1770                                          .585
 2270                                          .611
 2040                                          .227
  775                                          .227
 2080                                          .297
 1615                                          .580

 *** LEARNER VARIABLES - MEAN, VAR, AND STD. DEV. ***
VISUAL NUMERICAL   HAS MEAN =    .59  VAR. =        .46  SD. =     .68
AUDITORY NUMERICAL HAS MEAN =   -.29  VAR. =        .33  SD. =     .57
```

```
GROUP # 5, SKILL # 4  TOPIC 56
NUMBER OF STUDENTS RECOMMENDED =  10
------------------------------------------------------------------------
STUD #              STUDENT NAME              DISTANCE

                                                .923
 3075                                          1.283
 1375                                          2.014
 2740                                           .568
 1470                                          1.914
 0920                                          1.389
 3190                                          1.295
 1780                                          1.066
 1830                                           .568
 1860                                          1.283
 1870

 *** LEARNER VARIABLES - MEAN, VAR, AND STD. DEV. ***
VISUAL NUMERICAL   HAS MEAN =   -.15  VAR. =      1.44  SD. =   1.20
AUDITORY NUMERICAL HAS MEAN =  -1.49  VAR. =       .29  SD. =    .54
```

```
OMISSIONS GROUP

STUD #              STUDENT NAME              REASON

1360                                  NOT ELIGIBLE FOR ANY SKILL
1380                                  NOT ELIGIBLE FOR ANY SKILL
1390                                  NOT ELIGIBLE FOR ANY SKILL
1420                                  NOT ELIGIBLE FOR ANY SKILL
2170                                  NOT ELIGIBLE FOR ANY SKILL
1490                                  NOT ELIGIBLE FOR ANY SKILL
1740                                  NOT ELIGIBLE FOR ANY SKILL
1500                                  REMOVED FROM GRP # 2 DUE TO SIZE CONSTRAINTS
2240                                  NOT ELIGIBLE FOR ANY SKILL
1790                                  NOT ELIGIBLE FOR ANY SKILL
1570                                  NOT ELIGIBLE FOR ANY SKILL
 793                                  NOT ELIGIBLE FOR ANY SKILL SELECTED BUT
                                      ELIGIBLE FOR SKILLS 58
2370                                  NOT ELIGIBLE FOR ANY SKILL

NUMBER OF STUDENTS =  13
```

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